ALLOCATION OF COMMODITY FLOWS IN REGIONAL INPUT-OUTPUT TABLES FOR THE CZECH REPUBLIC

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Abstract

Regional input-output tables provide a detailed insight into the partial structure of the national economy. For a complete input-output analysis on the level of regional structures, it is necessary to know how these regions are interconnected among each other. Knowledge of this interconnectedness (i.e., import and export) is an important basis for complex models, which consider the overflow of effects between individual regions. Dependency on regions significantly influences overall results of an input-output analysis since effects may be spread across the country. In this article, two new methods for calculating interregional flows between regions are suggested. The third method proposed is based on self-adjusting of gravitational methods. The aim of this article is to demonstrate utility of these three methods on data for fourteen regions of the Czech Republic. All computations are provided by data based on estimated regional input-output matrices for the Czech Republic, compiled by the Department of Economic Statistics of the University of Economics in Prague for the year 2011. The final estimation of interregional commodity flows should serve as a useful source of information for planning and macroeconomic modelling of regional economies.

Key words: *regional input-output analysis, import-export, input-output tables, regionalization.*

1. Introduction

Input-Output analysis (I-O) is already a basic analytical tool used especially for studying the flows between products and the interaction of these flows between the final use elements. The SIOT table itself is a concept that allows these two sides of the equation to be shown in a single table that has a very elaborate structure while portraying a balanced state of these indicators.

The main application of this is primarily in economic policies, such as analyses on the impact of investments, tax rate changes or specific products' aggregate demand shocks. Input-Output analysis is able to provide an evaluation of the impacts on the main macroeconomic indicators. It is an important way of applying the aforementioned methods for the evaluation of impacts on a regional level.

Regional Input-Output tables provide information about the structure of the source side and the use in the regional segmentation context. These tables carry the same database as the national I-O table does, however, with a division for partial land units. The national table represents a hypothetical sum of regional Input-Output tables. The advantage of the regional I-O analysis is a more precise modelling of the impacts with an emphasis on regional differences. This analysis is especially applicable in cases where the economic influence in context with the regional specifics needs to be analysed. These specifics mean, for example, the closing of a mining factory in Moravian-Silesian country. The regional Input-Output analysis then shows detailed information about the structure of the impacts, not only on regional macro aggregates, but also on the employment rate, etc. In the case that the regional Input-Output tables (RIOT) carry the data about the import and export into different regions allocation, it is possible to combine these tables into one closed national entity. The resulting analysis allows not just the tracing of the impacts of the higher sort in the observed region, but also the analysis of the impacts on products in other regions.

RIOT tables are usually more difficult to obtain than the national SIOT tables. Most statistical offices do not make them, due to the detailed database and structural difficulty. In the Czech Republic, we have the SIOT tables available in five-year intervals (as well as the year 2009, due to the economic crisis), but RIOT tables are not officially created at all. Because of this, the Input-Output analysis methods for regionalization of the SIOT table were developed. However, connecting the individual regions remains to be completely solved.

In this paper, I will discuss the methods that allow the classification of the interregional import and export flows. Firstly, I will present the current state of knowledge, and in the subsequent chapter, I will depict the available data sources. This chapter is followed by a methodological section stemming from those available data sources. In the analysis, I will apply the above-mentioned techniques and I will discuss the differences in the final section of the paper.

2. Literature Review

The current methods of regionalization of I-O tables and, thus, the information about import and export mainly lie in three distinct approaches. The first approach lies in survey methods (Miller and Blair, 1985); the second, in the use of information about the regional structure of production (or employment rate) to regionalize the SIOT table; and the third approach is a combination of the first two (Flegg and Webber, 1997, 2000).

The novelty of regional Input-Output matrices is acknowledged in several studies that deal with the applications of I-O analysis in the Czech Republic. Additionally, we can mention the analysis (Šafr et al., 2013) that deals with the modelling of regional impacts in the Moravian-Silesian region. With regards to the unavailability of data sources (RIOT tables), the I-O analysis cannot be directly applied to the Czech data; thus, it is necessary to first obtain the tables. The choice of method of regionalization of the I-O matrix lies largely in the available data sources. Most approaches use so-called allocation coefficients (Bednaříková, 2012; Šafr et al., 2013). However, in the case of the Czech Republic, the approach has to be altered (there is no data on regional structure, though eventually gross value added is available) and substituted with the regional structure of employment in the calculations. However, as some studies suggest (Šafr, 2014), this approach tends to destabilize the RIOT matrices when the Czech database is used. The import and export is then constructed indirectly, making import/export a linear function of the gross value added (GVA) and End usage (EU). The structure of this calculation does not allow classifying the structure of import and export to other regions. This method then has two disadvantages - Import/Export is linearly dependent variables on the GVA/EU and does not allow distinguishing between the national (regional) import/export and international import/export outside of the Czech Republic. The department of Economic Statistics of the University of Economics conducted this research. This RIOT tables provide 82 products with a distinction between regional and international import/export (Sixta and Vltavská, 2016). Significant improvement in the context of the Czech Republic is found in the information about regional import and export of goods, which allows a wider amount of mathematical applications.

If we use this information, we can then connect the resulting regional Input-Output tables on the basis of the mathematical methods. Leontief and Strout (1963) have already formulated the first mathematical processes and conditions for the calculation of these interregional methods, especially the widely discussed Gravitational approach. The first applications of this approach for the Czech Republic have been provided by Kieslichová (2016); however, this approach is applied on aggregated flows between regions. It is based mainly on the application of regressive models for the estimate of production elasticity coefficient estimate with the aid of the RAS method[1] (Sargento et al., 2012).

The tables used were constructed during research at the University of Economics (Sixta et al., 2014; or Sixta and Vltavská, 2016) and the compiled RIOTs present the basis for further works, where the available data is not only about the regional RIOT table, but also addresses the volume of the regional export to other regions of the Czech Republic (but without specifying to which region). This work follows up on the research of the constructors of these tables, especially with the adjustment of the methods for available data sources. One of the main attributes of these tables that I continue to use is the recording of every region in every single product, covering both importer and exporter (not the aggregate, but single products).

3. Methodology

3.1 Basic Input-Output Methodology (Non-regional Framework – National Level)

The Input-Output table is divided into four quadrants, each covering a distinct part of the Input-Output analysis (intermediate consumption – matrix *X*), and, in accordance with the processing inputs, the intersectoral relations matrix of the first quadrant expresses the production flow between individual sectors/products of the national economy. The final consumption of individual components (consumption of households, state, and non-profit institutions) are represented in the second quadrant, as are the gross capital production and export (all as matrix *Y*). The added value of wages, taxes, surpluses, and capital consumption, as well the pure wage and import, when described by the Input-Output table, are in the third quadrant (as matrix *Z*). The basic relationship formula for base balancing the I-O equation is the following:

$$\sum_{j}^{n} x_{ij} + y_i = x_i \,, \tag{1}$$

where the x represents intermediate consumption from sector i to sector j. The y represents final consumption in sector i. The sum of those are total product (x). There are some conditions simplifying the reality of the I-O models are constructed on so that they can be analysed by a mathematical-statistical apparatus. These basic equations (1-5) describe the Input-Output analysis. We can assume that the individual inputs are perfect (Leontief, 1963) complements for each other, since the most vital condition (constant ratio of inputs in the production phase) is met for the needs of this equation.

[1] RAS method is based on multiplication of matrices: R, A and S. (Miller and Blair, 1985).

Leontief's production function is formulated:

$$x_{ij} = a_{ij} x_j, \tag{2}$$

because the technological coefficients are:

$$\mathrm{A} = (a_{ij})_{nn}, \text{where } a_{ij} = \frac{x_{ij}}{x_j}. \tag{3}$$

Furthermore, we can define the technological coefficients for matrix *Z*:

$$\mathrm{B} = (b_{ij})_{nn}, \text{where } b_{ij} = \frac{z_{ij}}{x_j}, \tag{4}$$

$$\sum_{i=1}^{l} b_{ij} + \sum_{i=1}^{n} a_{ij} = 1, \tag{5}$$

where a_{ij} are technological coefficients, which formulate how large a quantity of production from sector *i* is required for the generation of one component *j*, and b_{ij} are the amount of added value on one component generated in sector *j*. This applies to both coefficients: $0 \le a_{ij} \le 1, 0 \le b_{ij} \le 1$.

It is not possible to analyse individual subjects entering and exiting the market. The Input-Output analysis limitedly considers low production produced by markets and companies in individual sectors, which are seen as indefinitely large amount of subjects. We can write (in matrix-vector expression) the basic analytic equation, given the existence of inversion matrix, as:

$$x = (I - A)^{-1} y. \tag{6}$$

Traditionally, the I-O analysis assumes that imports and exports are parts of the Input-Output tables. In general, imports and exports can be entered as part of a second (or respectively third) quadrant in the case of imports. This is applied to the national I-O table and equally so in regional tables.

3.2 Inter-regionalism of the I-O Model

We can analogically define the formulas and variables for the regional Input-Output model for the region *R* ($R = 1, 2, \ldots, m$):

$$\sum_{j}^{n} x_{ij}^{R} + y_i^R = x_i^R, \tag{7}$$

$$\mathrm{A}^R = (a_{ij}^R), \text{ where } a_{ij}^R = \frac{x_{ij}^R}{x_i^R}, \tag{8}$$

$$\mathrm{B}^R = (b_{ij}^R), \text{ where } b_{ij}^R = \frac{z_{ij}^R}{x_i^R}, \tag{9}$$

where each *R* means the specific region's value in the case where we have *m* regions.

In simplified form, the relation between the national and regional I-O can be shown for the technical coefficients matrix:

$$\left[\sum_{R=1}^{m} \mathrm{A}^{R} diag(\mathrm{x}^{R})\right] diag(\mathrm{x})^{-1} = \mathrm{A}\,. \tag{10}$$

For regional export from region *R* to region *i* (I have marked the flow as "*R,i*", the first one refers to the region from and the second one to the region to. For export, it would be : $E_j^{R,i}$) of the product *j*, it can be written that:

$$\sum_{i=1}^{m} E_j^{R,i} = E_j^R,\ \sum_{R=1}^{m} E_j^R = E_j\,, \tag{11}$$

and analogically regional import, from region *P* ($P = 1,2,\ldots,m$) to any other region (*i*):

$$\sum_{i=1}^{m} I_j^{P,i} = I_j^P,\ \sum_{P=1}^{m} I_j^P = I_j\,, \tag{12}$$

for which applies:

$$I_j = E_j \text{, but it does not have to be } I_j^P = E_j^R\,, \tag{13}$$

which means that the total export at national level (sum of export of each region) of sector *j* must be same as the total import at national level from regions from sector *j*. But, the second equality says that the import from one region does not have to be (and it is mainly different) the same as the export from the second region (*R*), in the case where there are more than two regions. In the case where there are only two regions, it would be same. I would like to note that the flow from region *R* to region *i* can be viewed in two ways, although it is just one and the same number:

$$I_j^{P,R} = E_j^{R,P}\,. \tag{14}$$

In other words, the volume of regional export must be equal to the volume of regional import at the national (not regional) level. In this consideration, I have abstracted it from international import and export. I do not need it for my calculations, and, just in case it is seen in the same way as the import/export at the national level. Thus, the international export is set in the RIOT table, and it is not necessary need to calculate it anymore (without the country of destination).

3.3 Regionalization of Import and Export

The classification of import and export into certain regions can be done via a wide range of methods. In this work, I am concerning myself only with the methods that stem from the known database. In order to maximize the usage of the available data (and their specifics), I have proposed two possible solutions (3.1 and 3.2) that I compare with the generally used gravitational model approach (3.3).

3.3.1 Proportional Approach (RPA)

The proportional approach (RPA) stems from the assumption of unconditional probability and frequency approach. If it is assumed that the highest probability that the commodity will be exported from a certain region (*x*) to region (*y*), it is set by the volume of export (*x*) and import of that commodity (*y*) on the total export (or import) of that commodity. The following is the classification formula for the movement of commodities:

$$I_j^{P,R} = E_j^{R,P} = \frac{(\sum_{i=1}^{m} E_j^{R,i})(\sum_{i=1}^{m} i_j^{P,i})}{\sum_{R=1}^{m}\sum_{i=1}^{m} E_j^{R,i}} = \frac{E_j^R I_j^P}{E_j}. \quad (15)$$

The advantages of this approach are that:

- It has simple applicability of calculations.
- It ensures the consistency of the sum of lines and columns.
- It can be proven, in the case of our data structure, that the self-import to own region condition is automatically valid as defined by the formula.
- It tends to be the "middle road" solution.
- It proportionalizes the centre of requests and demands.

The disadvantages of this approach are that:

- The are conditions of proportional division of import and export.
- The regional costs of import/export between distant regions are not taken into account.
- It is necessary to have import and export with the same pricing.
- It does not abide by the natural structure of economy (market chains and product specifics).

3.3.2 The Optimalization Approach: Karush-Kuhn-Tucker Conditions (R-KKT)

Next approach is the optimalization approach based on Karush-Kuhn-Tucker conditions (R-KKT). The main disadvantage of the first 3.3.1 approach (RPA) is that it ignores the distance between regions. If we assume that the main factor in the commodity allocation between regions is distance, we can minimalize the objective function of import (or export) costs as:

$$\min\ f(I) = \sum_{P=1}^{m}\sum_{i=1}^{m}(\overline{w^{P,i}} I_j^{P,i}). \quad (16)$$

The general presumption of this function is the non-negativeness of individual flows between regions (based on the KKT theorem – Kuhn and Tucker, 1951). The values $\overline{w^{P,i}}$ are distances between region P and i. Furthermore, the sum of exports to other regions from the region (x) must be equal to the value of export, and the import to other regions must be equal to the export. Then, we are able to formulate individual conditions for import and export from the region R as:

$$\begin{gathered} \sum_{i=1}^{m} E_j^{R,i} = \overline{E_j^R}, \text{and } \sum_{i=1}^{m} I_j^{P,i} = \overline{I_j^P}, \\ I_j^{P,R} = E_j^{R,P}, \\ E_j^{R,i} \geq 0,\ I_j^{R,i} \geq 0, \\ E_j^{R,R} = 0, I_j^{R,R} = 0. \end{gathered} \quad (17)$$

Where the variables $\overline{E_j^R}$, $\overline{I_j^P}$, $\overline{w^{P,i}}$ represent the known parameters of the model. It is important to uphold the condition that the sum of regional import be equal to the sum of regional export; otherwise, the equation will not have a solution. It can be shown that, under the presumption of Equation (17), the model m of regions will contain $2m$+m of restrictions with $m \times m\text{-}m$ variables and $m \times m\text{-}m$ parameters of distance (weight) of the model and $2 \times m$ parameters of the volume of import and export (all only for the sector *j*).

The advantages of this approach are that:

- It is easy to apply and customize weights.
- There exists the possibility of penalization of certain trade routes (in the effort to get closer to reality).
- Through the multiplication of weights with the correction cell, we can adjust the cost of import and export and, as such, the distance of import and export between regions.

The disadvantages are that:

- The wrong weight setting can lead to overestimating the influence of cost of import and export on the volumes of import and export themselves.
- The automatic application of pure distance weights leads to not respecting natural economic structures (market chains and commodity specifics).
- It is impossible to calibrate the weights ex post (unknown regional structure of the commodity movement).

3.3.3. Newton's Gravity Model (NGM)

Newton's gravity model (NGM) stems from the assumption that the destination lies in the fact that the flows of commodities can be viewed as functions of regional production (consumption) and the distance between cities. As the name suggests, the construction stems from Newton's Law of Gravity, where, instead of the weight of the objects, their production (consumption) is taken into account, and, instead of distance, there is range. The gravitational constant influences the relative size of the flow and is called the proportionality constant. On this assumption, we can partially compare Newton's gravity model to the first (3.1) approach on the basis of probability. The resulting flow structure of the production between regions is set by a simple formula:

$$I_j^{R,P} = G_j \frac{(X_j^R)^\alpha (X_j^P)^\beta}{\delta^\omega}, \tag{18}$$

that conveys the relationship between the strength of the total production of these cities in the context of distance (δ^ω) and the proportionality constant (*G*). For ensuring the right influence of regions, I have considered the elasticity of production of importer and exporter (X_j^R, X_j^P) and the parameter that depicts the disintegration of the weight of distance. In the case of known interregional flows, the unknown parameters (α, β, ω) are easily estimated. The problem is with unknown interregional flows of production (as in our case). Sargento et al. (2012) suggest a two-stage iteration process, where, in the first step, the initial status of these parameters is estimated via customization of unknown parameters to the value 1. Then, through iteration, it converges to an accurate solution by the RAS method. However, the authors admit that this process generates about a 40% error margin. The application to Czech data (Kieslichová, 2016) shows another oddity, namely, that the results easily lead to the fact that, if the sector that does not export products, only imports (and vice versa), it can, after the application of this model, export an unbalanced product. That is caused by a distinctly large production of this product compared to other (exporting) regions.

With regards to the results of other authors, I have decided to alter the formula by the logic of the construction, so that it better reflects the core of the issue while it fully uses and respects regional differences (Newton's gravity model based on import-export-NGM-IE).

$$E_j^{R,P} = G_j \frac{X_j^R X_j^P}{\delta^{\omega}}. \tag{19}$$

After this correction, the resulting model better reflects the idea of the gravitational model in the sense of attraction of regions between each other. The upper item in the fraction expresses the product of export from the region *x* of the product *i* and also expresses the demand for the product *i* in the region *y* through demanded import. With regards to E_j^R, an I_j^R depicting the resulting clash of weights $(X_j^R)^{\alpha}$ and $(X_j^P)^{\beta}$, realizes the volumes of import and export between regions:

$$\left[\overline{G_j} \frac{X_j^R X_j^P}{\delta^{\omega}}\right] = \left[\overline{\overline{G_j}} \frac{(X_j^R)^{\alpha}(X_j^P)^{\beta}}{\delta^{\omega}}\right], \tag{20}$$

and the level difference should display into the calculation of gravitational constant *G*. The model in its adjusted form uses more detailed segmentation of the structure of import and export in regional RIOT tables. Simultaneously with the data and formula construction, it is secured that the region that does not export (or import) the product will not export or import either in regional classification. After that process (Sargento et al., 2012), I have analogically proceeded with this adjusted formula and transferred it to the following form for the estimate of the proportionality constant (the gravity between regions):

$$E_j^{R,P} = G_j \sum_{i=1}^{m} \frac{E_j^R I_j^P}{\delta^{\omega}} \wedge \omega = 1 \Rightarrow G_j = E_j^{R,P} \left[\sum_{i=1}^{m} \frac{E_j^R I_j^P}{\delta^1}\right]^{-1}. \tag{21}$$

Subsequently, it carries out preliminary estimates of the regional structure of the export/import and the standard I-O RAS optimization is method applied. The necessity of the RAS method application stems from the export (or import) concept of Newton's gravity method – the other side (in our case, the import side of the equation) is unbalanced. That is caused by the effort of these equations to keep the sum of at least the export or import, and that is secured by the proportionality constant (*G*). Here, it can be noted that balancing proportionally is an appropriate method for the constants matrix, and, after that, there is no need for applying the RAS or other balancing method of nonlinear optimisation on the preliminary flows of production.

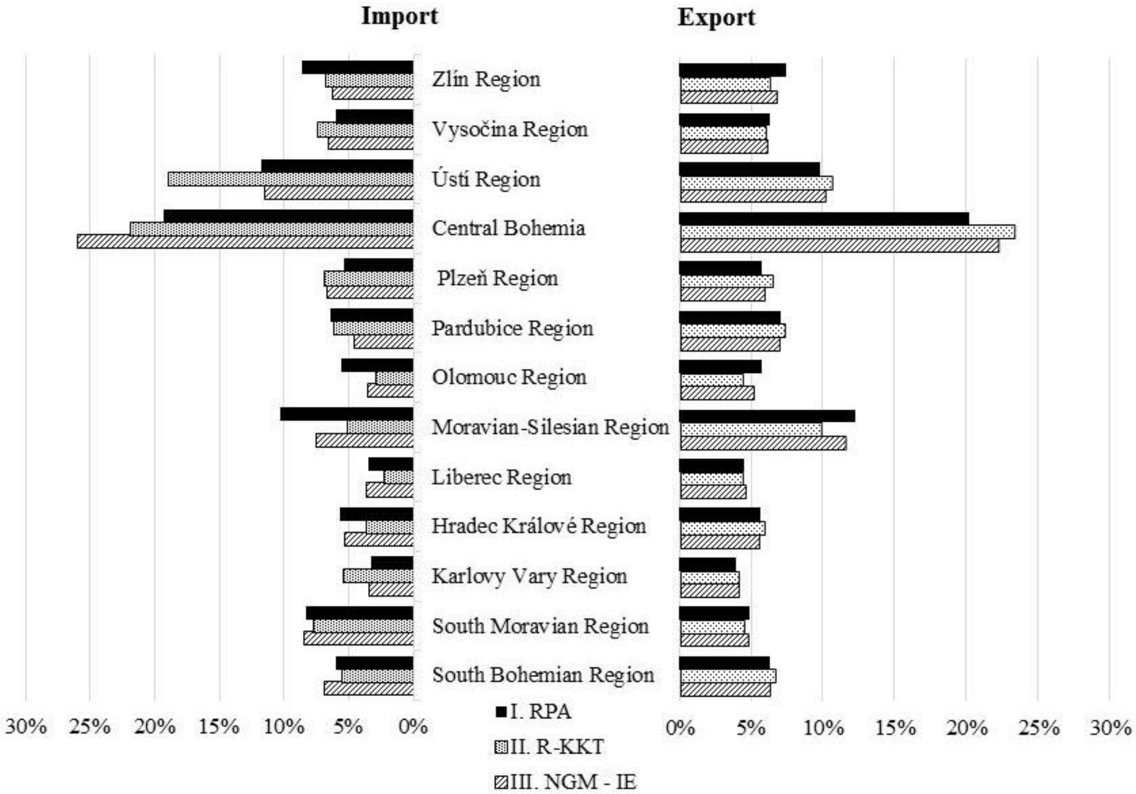
4. Results

After the application of the above-mentioned mathematical methods, I have estimated the production flows between fourteen Czech regions for 82 products. These methods then examined 29 848 possible mutual flows of production in the economy (29848=14 regions × 13 partners × 82 products × 2 import and export). Consequently, with the right aggregation of these flows, I can show data for every region. Because of the data volume, I have chosen one region and I have also focused on one of the problematic sections of the estimate for one particular (minor) product. I chose Prague for the comparison because there are a lot of products that Prague does not export and the composition is very specific. The same is valid for imports to Prague. Furthermore, I have focused on a flow analysis of services (CZ-CPA[2] 55-99). Services prove the highest amount of inconsistencies between the discussed methods and also show a detailed direct view of the financial products (CZ-CPA 64-66). Figure 1 shows the production flows from Prague to other regions.

[2] Classifications of Products (CZ-CPA).

All the results obtained by different methods are shown in the Table 1. Figure 2 describes import and export of services from Prague. Here, we see greater differences between R-KKT optimisation. In Figure 3, I have focused on the flow analysis of financial services. Prague is an important exporter of these services.

Figure 1: Graphic display of production flows to and from the Prague region



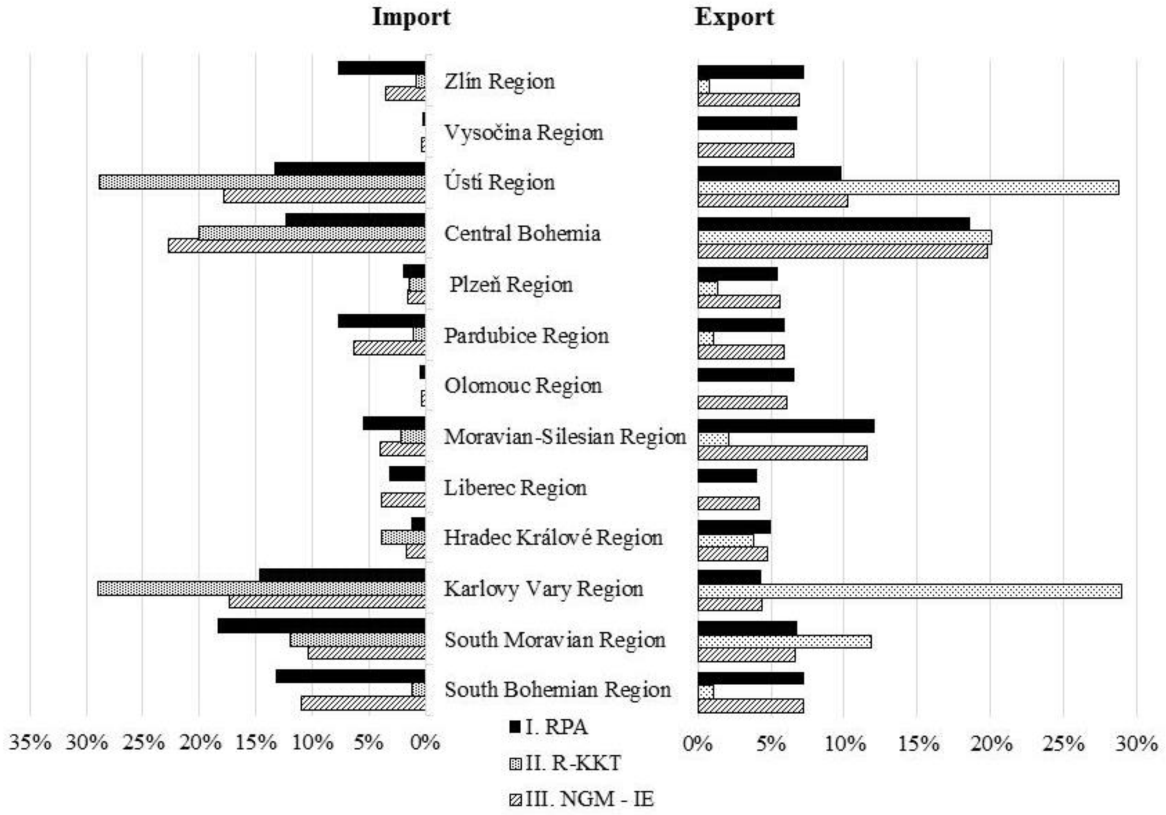
Source: the author.

Table 1: Aggregated production flows to and from the Prague region

Aggregate import / export for the Prague region						
	Import			Export		
Region \ Method	I. RPA	II.R-KKT	III. NGM - IE	I. RPA	II. R-KKT	III. NGM - IE
South Bohemian Region	6%	6%	7%	6%	7%	6%
South Moravian Region	8%	8%	8%	5%	4%	5%
Karlovy Vary Region	3%	5%	3%	4%	4%	4%
Hradec Králové Region	6%	4%	5%	6%	6%	6%
Liberec Region	3%	2%	4%	5%	4%	5%
Moravian-Silesian Region	10%	5%	8%	12%	10%	12%
Olomouc Region	6%	3%	3%	6%	4%	5%
Pardubice Region	6%	6%	4%	7%	7%	7%
Plzeň Region	5%	7%	7%	6%	6%	6%
Central Bohemia	19%	22%	26%	20%	23%	22%
Ústí Region	12%	19%	11%	10%	11%	10%
Vysočina Region	6%	7%	7%	6%	6%	6%
Zlín Region	9%	7%	6%	7%	6%	7%

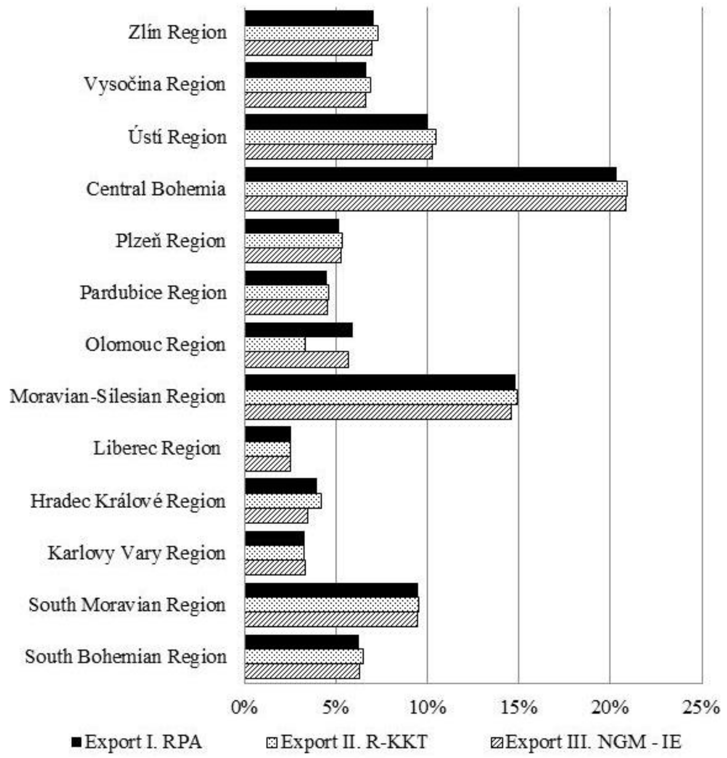
Source: the author.

Figure 2: Import and export of services in the Prague region (CZ-CPA 55-99)



Source: the author.

Figure 3: Export of financial services in the Prague region (CZ-CPA 64-66)



Source: the author.

5. Discussion

The aforementioned three methods yield very similar results. Considering the unknown structure of imports and exports of individual regions, it is problematic to discuss the successes of the classifications of these methods. However, the results point to the conclusion that, on the aggregate level of Prague (and also different regions), all methods converge to similar production flow classification into other regions. The Central Bohemian region, Ústí region, and Moravian-Silesian region appear as the most important business partners to Prague. This is confirmed through all three methods, wherein the accordance of results through all methods regarding the flows can be seen in other regions as well - not just in Prague.

From the viewpoint of import and export, the most problematic aspect is the classification of services. We can say that services are not as demanding transport-wise as hard goods (where they form the larger part of the value). This attribute of services affects the solution through the minimization of objective function of transport expenses (R-KKT). This problem is shown in the results for services as a whole on the figure of import and export in Prague. Prague is a region that mainly provides services to the Czech market throughout all regions. Through R-KKT, I have achieved, for the export of services, completely different estimates of the flow classification than through the other two methods. It can be stated that the RPA and NGM-IE methods converge with their results to each other, both on national and regional levels. That is mainly due to common main variables (import and export). The differences between RPA and NGM-IE are there mainly because NGM-IE considers a specific gravitational constant and, for every region, a distance factor. Furthermore, in NGM-IE we need to apply the nonlinear RAS method for balancing the sum of lines and columns. The difference between the RSA and the NGM-IE on one side and the R-KKT method on the other side is expressed due to the mentioned reasons. On the national level, these products are not as demanding on transfer.

This issue does not occur in the financial products. That is mainly because Prague is the main exporter of these products, and there is no space between the individual methods for significant differences to be shown. It means that production is allocated very similarly.

The aforementioned preliminary results can be compared on the Czech level particularly with the work of Kieslichová (2016), although she used the Newtonian method without the alteration for imports and exports (meaning in its original form). This comparison of regional imports and exports shows that the results do not unbalance the RIOT tables. I have also avoided such an effect that the regions, which do not export products, are not counted after the application of the method. That is precisely the effect of the usage of regional data as the reason for improving the Newtonian method from production to imports and exports.

6. Conclusion

The aim of this article was to demonstrate utility of three methods to inter-regional flows allocation in regional input-output tables. Several possible options how to classify or count these flows can be found in the literature. Most of those approaches are based on companies' data sets and, therefore, can be applied only on some specific cases under certain conditions. In this context, I have illustrated two very different and easily applicable (with different characteristics for end data) approaches for the classification of product flows between regions. I have used these two new approaches directly in the context of our available database for the Czech Republic. For comparison, I have added a modified Newtonian model.

The results show surprising similarity between all methods used. Most similarities appear for the results of the RPA and NGM-IE methods. In the NGM-IE, I have taken into account, apart from imports and exports, the distance variable, gravitational constant for every product type, and the RAS method; meanwhile, the RPA method is simply a calculation based on the frequency of unconditional probability. The method of application of the Karush-Kuhn-Tucker optimisation theorem on the regional level (R-KKT) appears similar to these two methods as well. However, more significant differences are reached mainly in services (illustrated). This result can be explained by the fact that the regionalisation of services is not as largely determined by distance compared to other products.

It is arguable to say that the newly introduced methods allow for better classification of product flows compared to the Newtonian method in its basic form. However, it can be said that all the methods achieve very similar results and avoid some issues that are present in the calculation (for example, in Kieslichová, 2016; Sargento, Ramos and Hewings, 2012). The results presented should be considered preliminary. Further development in this field will be focused on encompassing the distance into the RPA method and improving the estimates of the objective function weights and other factors influencing the classification (in the R-KKT method). Also, it is arguable to set the quadratic optimization for reaching more robust estimates.

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