

## **REDISTRIBUTIVE EFFECT AND PROGRESSIVITY OF TAXES – COMPARISON ACROSS DIFFERENT TYPES OF FAMILIES IN POLAND**

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### **Abstract**

*The demographic problem, which consists not only in unfavourable changes in the population structure in terms of age, but also in the declining fertility rate, currently affects numerous countries, including Poland. As observed since 1992, the fertility rate in Poland has not guaranteed a simple replacement of generations. In the context of demographic threats it is crucial to implement an appropriate pro-family policy. Financial issues are pointed out as one of the main reasons for a low or negative population growth rate. The paper focuses on analyzing the tax system in Poland in the context of helping families. The main purpose of the article is a statistical analysis and a comparison of the redistributive effect of personal income taxes in the 4 groups or types of families. The approach is applied in the analysis of fairness of the income-tax system when the population is partitioned into heterogeneous groups of families.*

**Key words:** *personal income tax, structural progression, tax fairness, reranking.*

### **1. Introduction**

The demographic problem, which consists not only in unfavourable changes in the population structure in terms of age, but also in the declining fertility rate, currently affects numerous countries, including Poland. As observed since 1992, the fertility rate in Poland has not guaranteed the simple replacement of generations. In the context of demographic threats it is crucial to implement the appropriate pro-family policy, especially because demographers forecast the further escalation of these tendencies. Taking into account the above-mentioned threats it is important to provide support for families as regards the costs of maintaining and upbringing children. The state may forward the activities undertaken by families by means of the appropriate tax system. It is crucial to apply tax system solutions that help taxpayers with low income or families with many children, prevent dependence on welfare aid and, concurrently, are fair. The reconciliation of all these objectives is not easy and requires regular monitoring of the effects of implemented solutions.

It is an unquestionable fact that the structure ensuring a fair fiscal mechanism should be an inherent characteristic of every tax system. Justice is the value of utmost importance in the contemporary society. The achievement of tax fairness is very difficult as no readily available and universal methods or the way for its implementation have been formulated.

The main purpose of the conducted research was to assess and compare the fairness of the Polish tax system between groups of taxpayers distinguished by family type. Apart from the fairness of the tax system, the research compared the effects of income redistribution and tax

progressivity. In this paper the equity in the income tax system is defined by means of three axioms introduced by Kakwani and Lambert (1998). The method described by Monti et al. (2015) was used for evaluation of the contribution of each type of family to the overall inequity, and for judgment of how violations of axioms discriminate among type of family groups in their reciprocal relationships. The method allows to reveal how a tax system distributes its possible inefficiencies through different groups of income units. The analysis was conducted on the basis of real data regarding the revenue of taxpayers filing a joint tax return in the selected tax office in Wrocław.

## 2. Methodology

The decomposition of the redistribution index is one of the possibilities of assessing the unfairness of income tax. In 1998, the notion of fairness of a tax system was defined mathematically by Kakwani and Lambert (1998) through the prism of three postulates that should be fulfilled by a fair tax system. If a tax system fails to fulfil even one of these postulates, it becomes unfair. Horizontal and vertical equity are two of the basic commands of social justice that have been applied to income tax and many authors consider the fairness of the tax system through the violation of these basic commands (e.g. Aronson and Lambert, 1994; Aronson et al., 1999; van de Ven et al., 2001; Urban and Lambert, 2008).

Let  $x_1, x_2, \dots, x_n$  represent income before taxation (gross) for  $n$  taxpayers who pay the amount of tax of  $t_1, t_2, \dots, t_n$ , respectively, where the tax paid  $t$  is a function of income, which can be generally expressed as  $t(x)$ , i.e.  $t_1 = t(x_1)$ ,  $t_2 = t(x_2)$ ,  $\dots$ ,  $t_n = t(x_n)$ . Then,  $y_i = x_i - t_i$  will represent income after taxation (net) and  $a_i = t_i / x_i$ , the tax rate for the  $i$ -th taxpayer.

Postulate 1 – the principle of minimum progressivity stipulates that a tax to be paid should increase monotonically with payment capabilities. It means that people earning higher income should not pay a lower tax, which can be formally described as follows:

$$\text{if } x_i \geq x_j \text{ then } t_i \geq t_j \quad (1)$$

Weak inequalities observed in the above implication make it possible to treat the horizontal justice as a special case of the principle of minimum progressivity.

Postulate 2 – the principle of progressivity stipulates that wealthier taxpayers should not only pay a higher tax, but they should also return a larger part of their income, which is defined by the following formula:

$$\text{if } x_i \geq x_j, t_i \geq t_j \text{ then } \frac{t_i}{x_i} \geq \frac{t_j}{x_j} \quad (2)$$

According to postulate 2, the richer people must pay taxes at higher rates. Of course, a violation of minimal progression automatically entails a violation of this principle. A breach of the principle of minimum progressivity automatically violates the second principle of progressivity. Furthermore, weak inequalities in the above implication indicate that a proportional tax is fair according to the presented principles of fairness.

If a tax system infringes both the first and second principles, it means that the system is regressive, i.e. the higher income is earned by a taxpayer, the lower income tax is paid.

Postulate 3 – the principle of preserving the order stipulates that the system which fulfils the first and second principles should not cause any changes of position in the ranking of taxpayers in terms of their earned gross income. It means that if a given taxpayer was in a better situation (i.e. such taxpayer earned higher gross income) in relation to another taxpayer

before taxation, this taxpayer should remain in a better situation also after taxation, i.e. such taxpayer's net income should also be higher. The third principle can be represented as the following formula:

$$\text{if } x_i \geq x_j, t_i \geq t_j \text{ and } \frac{t_i}{x_i} \geq \frac{t_j}{x_j} \text{ then } x_i - t_i \geq x_j - t_j. \quad (3)$$

The third principle is identified with vertical justice, limited, however, to the case of retaining the position in the ranking of taxpayers' wealth with respect to their income before and after taxation.

The measures that allow to identify rerankings are based on the Gini coefficient and on concentration coefficient. Concentration coefficients are calculated similarly as Gini coefficients, except that a different criterion is applied to ascending sorting of data.

Let  $I$  denote a net income or a tax. Then the concentration coefficient of income after tax distribution or tax distribution,  $I = Y, T$  is defined by the formula:

$$D_I = 1 - 2 \int_0^1 C_I(p) dp \quad (4)$$

where  $C_I(p)$  is a tax concentration curve –  $C_T(p)$  or a net income concentration curve –  $C_Y(p)$ . For a finite population of  $n$  taxpayers, a *tax  $T$  concentration curve* is defined as a continuous, piecewise linear function, with vertices in points  $(p_i, C_T(p_i))$ ,  $i = 0, 1, 2, \dots, n$ ,

where  $p_0 = 0$ ,  $C_T(p_0) = 0$ , and for  $i = 1, 2, \dots, n$ ,  $p_i = \frac{i}{n}$ ,

$$C_T(p_i) = \frac{\sum_{j=1}^i t_j}{\sum_{j=1}^n t_j} = \frac{1}{n\mu_T} \sum_{j=1}^i t_j, \quad (5)$$

where  $\mu_T = \frac{1}{n} \sum_{j=1}^n t_j$ .

A net income (after tax) concentration curve is analogously defined as a piecewise linear function with vertices in points  $(p_i, C_Y(p_i))$ ,  $i = 0, 1, 2, \dots, n$ , where  $p_0 = 0$ ,  $C_Y(p_0) = 0$ , and for

$i = 1, 2, \dots, n$ ,  $p_i = \frac{i}{n}$ ,

$$C_Y(p_i) = \frac{\sum_{j=1}^i y_j}{\sum_{j=1}^n y_j} = \frac{1}{n\mu_Y} \sum_{j=1}^i y_j, \quad (6)$$

where  $\mu_Y = \frac{1}{n} \sum_{j=1}^n y_j$ .

The measurement of tax unfairness is related to the measurement of redistributive effect. For the purposes of this paper, redistribution consists in the state's intervention, through a tax system, in income inequalities. Therefore, the measurement of the level of redistribution involves the comparison of coefficients measuring the extent of inequalities in the distribution of income before and after its taxation.

Based on the three postulates of a fair tax system (PIT), defined by the relations given in (1)-(3), the fairness of a tax system can be assessed, and it is possible to estimate the extent of loss in income redistribution caused by a violation of respective principles of justice. The following decomposition of the redistribution index should be used for the purpose:

$$RE = V - S_1 - S_2 - S_3, \quad (7)$$

where  $S_1$ ,  $S_2$  and  $S_3$  estimate the loss in redistribution resulting from the violation of fair taxation postulate 1, 2 and 3, respectively, and  $V$  estimates the potential level of redistribution that could have been achieved, if the tax system did not violate the principles of fair taxation. If  $G_Y$ ,  $G_T$ ,  $G_{\frac{T}{X}}$  denote the Gini coefficients for income after taxation, tax and tax rate, respectively, and  $D_Y$ ,  $D_T$ ,  $D_{\frac{T}{X}}$ , the concentration coefficient for income after taxation, tax and tax rate, respectively, then  $S_1$ ,  $S_2$  and  $S_3$  in formula (7) can be calculated in the following manner:

$$S_1 = \frac{\sum_{i=1}^n t_i}{\sum_{i=1}^n (x_i - t_i)} \cdot R_T, \quad (8)$$

$$S_2 = \frac{\sum_{i=1}^n t_i}{\sum_{i=1}^n (x_i - t_i)} \cdot (R_{\frac{T}{X}} - R_T), \quad (9)$$

$$S_3 = R_Y, \quad (10)$$

where:

$$R_Y = G_Y - D_Y, \quad R_T = G_T - D_T, \quad R_{\frac{T}{X}} = G_{\frac{T}{X}} - D_{\frac{T}{X}}.$$

$R_Z$ ,  $Z \in \left\{ Y, T, \frac{T}{X} \right\}$  is the re-ranking index, since, in accordance with the definition of the

Gini coefficient and the concentration coefficient, this index can assume a value from the interval  $[0, 2 \cdot G_Z]$ . A zero value means that taxpayers ranked in respect of the value of the characteristic  $Z$  retained their positions held in the ranking of taxpayers with respect to income before taxation. The more shifts occur in the ranking of taxpayers in respect of the value of the characteristics  $X$  and  $Z$ , the higher is the value of the index  $R_Z$ . For a discussion of the measures, see (Pellegrino and Vernizzi, 2013).

In accordance with formulas (7) – (10), the postulates of fair taxation are violated if taxpayers are ranked differently with respect to income before taxation and with respect to tax or tax rate or income after taxation. It means that a breach of the postulates occurs when  $S_1 > 0$  or  $S_2 > 0$  or  $S_3 > 0$ .

The analysed population is often inhomogeneous and, therefore, it is advisable to conduct an analysis within groups which are homogeneous in respect of a tax system, as well as a comparative analysis between such groups.

If an inhomogeneous population of taxpayers is divided into  $L$  homogeneous groups, it is possible to decompose the reranking index  $R_Z$ ,  $Z \in \left\{ Y, T, \frac{T}{X} \right\}$  (Monti et al., 2010; Ebert, 2010), owing to which the reranking within each respective group –  $R_Z^W$ , and the reranking between the distinguished groups –  $R_Z^{AG}$  can be assessed:

$$R_Z = R_Z^W + R_Z^{AG}. \quad (11)$$

With the use of the above decomposition, the direction and extent of the ranking between the groups in respect of  $Z$ ,  $Z \in \left\{ Y, T, \frac{T}{X} \right\}$  can be described (Monti *et al.*, 2015):

$$R_Z = \sum_{h=1}^L \sum_{j=1}^L (R_Z^{h,h} + R_Z^{h>j} + R_Z^{h<j}) \quad (12)$$

$R_Z^{h,h}$  depends on the reranking determined within the respective groups, whereas  $(R_Z^{h>j} + R_Z^{h<j})$ , on the reranking observed between the  $h$  and  $j$  groups. The term  $R_Z^{h>j}$  applies to the reranking of the individuals belonging to the groups  $h$  and  $j$ , in which the value of the characteristic  $Z$  for the individuals in the group  $h$  is higher than that for the individuals belonging to the group  $j$ . A reverse situation is represented by the term  $R_Z^{h<j}$ , where the values of the characteristic  $Z$  for the individuals belonging to the group  $j$  exceed the said values for the individuals from the group  $h$ .

### 3. Description of the data and results of the analysis

To present the nature of the information about the tax system in the context of a family, provided by the statistical analysis contained in this paper, the real data were used, concerning taxpayers submitting joint tax return to one of the Tax Offices in Wrocław.

The whole population of taxpayers was divided into homogeneous groups in respect of the so-called pro-family tax relief provided for in the tax system. Thus, four groups were distinguished:

- C – spouses filing a joint tax return and not making use of a child tax relief (54.5%),
- C+1 – spouses filing a joint tax return and making use of a tax relief for one child (28.0%),
- C+2 – spouses filing a joint tax return and making use of a tax relief for two children (15.1%),
- C+3 – spouses filing a joint tax return and making use of a tax relief for three or more children (2.4%).

The groups distinguished above will be referred to as tax families and the analysis will be conducted both for the whole population and by family type.

When applying income per capita, wealth of small households is overestimated compared to big households, as a result of relatively constant expenses (e.g. heating, TV), therefore one should apply equivalence scales. An equivalence scale is a parameter calculated to determine the nature of the influence exerted by a demographic composition of a household on its cost of living. The scale lets know how many times more or less must a given household spend (e.g. two adults and two children) so as to become as wealthy as a household of a different type (e.g. a one person household). The greater the share of spending for highly elastic goods (e.g. food, clothing) with respect to the number of persons, the greater the relevant multiplier

transforming the number of persons in a household into the number of equivalent (“standard”) persons.

In this paper, to ensure the comparability of incomes and taxes between the groups the nominal income was transformed into the equivalent income on the basis of the OECD scale, for which a married couple not making use of a child tax relief serves as a reference. The scale values for respective family types are presented in Table 1.

Table 1: The values of equivalent scale OECD

Characteristics of taxpayers submitting joint tax return		OECD scale value
Number of adults	Number of children	
2	0	1.0
2	1	1.2
2	2	1.4
2	3	1.6
2	4	1.8
2	5	2.0
2	10	3.0

Source: the author.

For equivalent income and tax values in the distinguished groups, the decomposition of the redistribution index  $RE$  was carried out in accordance with formula (7), and its results are presented in Table 2.

Table 2: Overall  $RE$  decomposition ( $\times 100$ )

Gini for pre-tax income	Gini for post-tax income	$RE$	Potential equity	Postulate 1	Postulate 2	Postulate 3	Postulates 1-3
36.13	34.03	2.10	2.68	0.15	0.41	0.02	0.58
as a percentage of $RE$ (%):		100	127.61	7.14	19.52	0.95	27.61

Source: the author.

The Gini coefficient ( $\times 100$ ) for income before taxation is 36.13, whereas as a result of taxation the inequality in the distribution of income after taxation drops to the level of 34.03. It means that the tax system reduces inequalities in income distribution by 2.1 percentage points. The potential level of the redistributive effect reaches 2.68 percentage points and it is the value by which the Gini coefficient could decrease if all instances of injustice resulting from the violation of the three postulates of fairness were eliminated. Hence, the instances of injustice in the tax system block a reduction in the level of income inequalities by additional 0.58 percentage points. The results given in Table 2 show also that the violation of the three postulates of a fair tax system represents 27.6 percent of the achieved level of redistribution. The smallest loss in the level of redistribution results from a breach of the third postulate, i.e. shifts in the positions occupied by taxpayers in the ranking in respect of their income before and after taxation lead to a loss in potential redistribution of 0.95% of the  $RE$  level. The biggest loss in potential redistribution is observed as a result of violating the second postulate, as it reaches almost 20 percent of the level of  $RE$ , whereas the severity of infringing the first postulate is at the level of 7.4 percent of  $RE$ .

The next step of the analysis focused on detecting the directions of violations of respective postulates for married taxpayers belonging to various types of families, as defined above. To

this aim the decomposition of the reranking index was used and the decomposition was carried out (12). Rerankings across two types of family, *A* and *B*, can be observed in two different situations presented in Table 3. For definiteness, the Table 3 presents reranking for tax.

Table 3: The directions of reankings

	Reranking 1		Reranking 2	
	Group A	Group B	Group A	Group B
gross income				
tax	$x_{A,i} < x_{B,j}$		$x_{A,i} > x_{B,j}$	
		$t_{A,i} > t_{B,j}$		$t_{A,i} < t_{B,j}$

Source: the author.

The reranking 1 favours group B, the second – group A.

Applying the described method we are able to determine the contributions to the losses of the redistributive effect due to outperforming of one group of the type family by the second. We can also find the contributions to the losses of the redistributive effect due to outperforming of one group of the type family by all the others types, and the contribution due to outperforming of each type families by the one type.

The results illustrating the directions and extent of violating the respective postulates of a fair tax system in the distinguished groups are collected in Tables 4-6.

Table 4: Comparisons of losses between group pairs for postulate 1  
 (% of the two contrasting directions)

$\frac{R_T^{h>j}}{R_T^{h,j}}\%$	<i>j</i>				
	C	C+1	C+2	C+3>	All groups
<i>h</i> C	-	86.64	91.32	93.09	89.31
C+1	13.36	-	68.19	78.81	26.22
C+2	8.68	31.81	-	67.12	14.32
C+3>	6.91	21.19	32.88	-	11.34
All groups	10.69	73.78	85.68	88.66	

Source: the author.

Table 5: Comparisons of losses between group pairs for postulate 2  
 (% of the two contrasting directions)

$\frac{R_A^{h>j} - R_T^{h>j}}{R_A^{h,j} - R_T^{h,j}}\%$	<i>j</i>				
	C	C+1	C+2	C+3>	All groups
<i>h</i> C	-	92.07	96.28	97.93	94.29
C+1	7.93	-	75.58	89.14	18.56
C+2	3.72	24.42	-	71.76	7.58
C+3>	2.07	10.86	28.24	-	4.64
All groups	5.71	81.44	92.42	95.36	

Source: the author.

Table 6: Comparisons of losses between group pairs for postulate 3  
 (% of the two contrasting directions).

	$\frac{R_Y^{h>j}}{R_Y^{h,j}}\%$	$j$				
		C	C+1	C+2	C+3>	All groups
$h$	C	-	18.36	14.60	13.44	16.22
	C+1	81.64	-	39.14	33.75	70.69
	C+2	85.40	60.86	-	43.39	79.10
	C+3>	86.56	66.25	56.61	-	80.10
	All groups	83.79	29.31	20.89	19.90	

Source: the author.

When spouses without children eligible for a tax relief are compared to spouses with three or more children eligible for a tax relief, it can be observed that when the first postulate is violated, 93.09% (Table 4) of the cases include spouses without children who pay an absolutely higher tax, whereas in as few as 6.91% of the cases a high tax is paid by families with three or more children. If these two types of families are compared in the case when the second postulate is violated, it can be observed that the system disfavours families without children to even a greater extent (Table 5) in comparison with families with three or more children. A breach of the second postulate is caused, in 97.93% of the cases, by a relatively high tax rate applicable to families without children. When a breach of the third postulate is considered, inverse relations between these groups can be observed (Table 6). More than 80% (86.56%) of the rerankings caused by a breach of the third postulate are to the advantage of families with three or more children, whereas only 13.44% of these rerankings favour families without children.

It can be observed that the postulates of fair taxation are violated by the Polish tax system in general to the advantage of families with children, and families with many children in particular.

On the basis of the results shown in the last column of Table 4, it can be concluded that as many as 89.31% of the reranking in terms of tax are to the disadvantage of married couples without children, whereas in as few as 11.34% of the cases when the first postulate is violated, families with many children are disfavoured.

The analysis was performed similarly as the analysis conducted for the Italian tax system that was found much more unfair (Blangiardo and Vernizzi, 2013; Monti et al., 2015).

#### 4. Conclusion

The analysis was performed with data from 2007, and, therefore, the results should be regarded as an initial research which consisted in investigating the possibilities of assessing the fairness of a tax system in the context of family offered by the proposed methods. It is planned to gather the latest data regarding incomes and the paid tax and to conduct again an analysis to assess the applicable tax system and to compare respective groups distinguished by family type.

On the basis of the conducted analysis it can be concluded that the Polish tax system did not lead to excessive injustice consisting in rerankings by distributions of income before and after taxation. A breach of the second postulate gave rise to the biggest loss at the level of income redistribution caused by a tax system.

In general, violations of the fair taxation postulates caused by the Polish tax system with regard to the analysed population of taxpayers are to the advantage of families with children,

and families with many children in particular. A breach of the first postulate disfavours families without children to the greatest extent, while it is the least disadvantageous to families with many children. The same relationships regarding the favouring of respective family types by the tax system can be observed when the second and third postulates are violated. The more children a family has, the more advantageous its treatment by a tax system is.

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