A NOTE ON ASADA'S MACRODYNAMIC MODEL WITH FIXED PRICES

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Abstract

Asada in 2014 set up and analyzed a two-dimensional macrodynamic model with fixed prices without active macroeconomic stabilization policy describing the development of firms' debt and the real national output. He showed that if at the equilibrium point of the model the derivative of its adjustment cost function of investment with respect to the function of investment and the derivative of the function of investment with respect to firms' debt are sufficiently large, then the Jacobian of the model at the equilibrium point has a pair of pure imaginary eigenvalues. In the present paper it is shown that in some cases this property need not be satisfied

Key words: *dynamical system, equilibrium, Jacobian matrix, eigenvalues.*

1. Introduction

Asada (2014) introduced two-dimensional macrodynamic model (1) with fixed prices which consists of the following two equations

$$\dot{d} = \phi(g(r, \rho, d)) - s_f \{r - i(\rho, d)d\} - g(r, \rho, d)d = f_1(d, y) \quad (1.1)$$

$$\dot{y} = \alpha[c + \phi(g(r, \rho, d)) + \nu - y] = \alpha f_2(d, y), \quad (1.2)$$

$$c = (1 - s_1)\{(1 - s_f \beta)y - \tau(y)\} + (1 - s_2)\, i(\rho, d)d + (1 - s_3)\rho b,$$

where D – stock of firms′ nominal private debt, p – price level, K – real capital stock, $d = D/(pK)$ – private debt-capital ratio, $g = \dot{K}/K$ – the investment function with the $\partial g/\partial r > 0$, $\partial g/\partial \rho < 0$, $\partial g/\partial d < 0$, $\phi(g)$ – adjustment cost function of investment that has the properties $\partial \varphi(g)/\partial g \geq 1$, $\partial^2 \varphi(g)/\partial g^2 > 0$ which was introduced by Uzawa (1969), P – real profit, $r = p/K$ – rate of profit, i – nominal rate of interest that is applied to firms′ private debt, ρ – nominal rate of interest of the government bond, Y – real output, $y = Y/K$ – output-capital ratio, $r = \beta y$, $0 < \beta < 1$, G - real government expenditure, $\upsilon = G/K$ –

government expenditure-capital ratio, υ – constant, B – stock of nominal government bond, $b = B/(pK)$ – government bond-capital ratio, b – constant, α – quantity adjustment speed of the disequilibrium in the goods market, C – real private consumption expenditure, $c = C/K$ – private consumption expenditure-capital ratio, T – real tax, $\tau = T/K$ – tax-capital ratio, and the parameters s_1, s_2, s_3, s_f move in the intervals $0 < s_1, s_2, s_f < 1$, $0 < s_3 \le 1$.

Suppose that there is an isolated equilibrium of model (1) (d^*, y^*). Jacobian of system (1) at (d^*, y^*) is

$$J(d^*, y^*) = \begin{pmatrix} f_{11} & f_{12} \\ \alpha f_{21} & \alpha f_{22} \end{pmatrix},$$

where

$$\begin{aligned} f_{11} &= \frac{\partial f_1(d^*, y^*)}{\partial d} = [\phi'(g) - d]g_d - g + s_f(i_d d + i), \\ f_{12} &= \frac{\partial f_1(d^*, y^*)}{\partial y} = \beta\{[\phi'(g) - d]g_r - s_f\}, \\ f_{21} &= \frac{\partial f_2(d^*, y^*)}{\partial d} = (1 - s_2)(i_d d + i) + \phi'(g)g_d, \\ f_{22} &= \frac{\partial f_2(d^*, y^*)}{\partial y} = (1 - s_1)((1 - s_f\beta - \tau_y)) + \beta\phi'(g)g_r - 1. \end{aligned} \tag{2}$$

For the existence of cycles around the equilibrium (d^*, y^*)it is necessary to have a pair of pure imaginary eigenvalues of Jacobian $J(d^*, y^*)$ (see for example Asada et al., 2006; Maličký and Zimka, 2010; Asada, 2012; Makovínyiová and Zimka, 2013; Sordi, 1986; Yukalov et al., 2009). The eigenvalues are the roots of the equation

$$\lambda^2 - TrJ(d^*, y^*)\lambda + \det J(d^*, y^*) = 0,$$
$$TrJ(d^*, y^*) = f_{11} + \alpha f_{22}, \ \det J(d^*, y^*) = \alpha(f_{11} f_{22} - f_{12} f_{21}),$$

and are determined by the formula

$$\lambda_{1,2} = \frac{TrJ \pm \sqrt{(TrJ)^2 - 4\det J}}{2}.$$

The necessary and sufficient conditions for pure imaginary eigenvalues are

1. $TrJ(d^*, y^*) = 0$,
2. $\det J(d^*, y^*) > 0$.

These conditions are satisfied if

$$f_{11} < 0,\ f_{12} > 0,\ f_{21} < 0,\ f_{22} > 0,\ f_{11} f_{22} - f_{12} f_{21} > 0. \tag{3}$$

Calculation of $f_{11} f_{22} - f_{12} f_{21}$ gives

$$\begin{aligned} f_{11} f_{22} - f_{12} f_{21} = &-(\phi_g(g) - d)\, g_d \{(1 - s_1)\tau_y + s_1(1 - s_f\beta)\} \\ &+ \beta\phi_g(g)\, g_r\{-g + (s_f + s_2 - 1)(i_d d + i)\} \\ &+ \{-g + s_f(i_d d + i)\}\{(1 - s_1)(1 - s_f\beta - \tau_y) - 1\} \\ &+ (1 - s_2)(i_d d + i)(s_f + dg_r)\beta + dg_d s_f\beta. \end{aligned} \tag{4}$$

Asada supposes on the base of the structure of the terms (2) and the structure of expression (4) that the inequalities (3) can be satisfied if $\frac{\partial\phi(g)}{\partial g}$ and $\left|\frac{\partial g}{\partial d}\right|$ are sufficiently large at the equilibrium $\left(d^*, y^*\right)$. In the next section it will be shown substituting in model (1) the general functions $\phi(\mathrm{g}), \mathrm{g}(\mathrm{r}, \rho, \mathrm{d}), \mathrm{i}(\rho, \mathrm{d}), \tau(\mathrm{y})$ by specific functions with relevant economic meaning that in some cases inequalities (3) cannot be satisfied.

2. Considerations on the fulfilling of inequalities (3)

Let us take for the general functions in the model the following functions:

1. $\phi(g) = ag^2, a > 0,$
2. $g = g(r, \rho, d) = \dfrac{\kappa}{1+e^q},\ q = md + o\rho - pr,\ \kappa, m, o, p$ – positive parameters, (5)
3. $i = i(\rho, d) = \rho + i_1 d,$
4. $\tau(y) = \tau_1 y - T_0.$

The function $\phi(g) = ag^2, a > 0,$ satisfies Uzawa conditions, and the function $g = \frac{\kappa}{1+e^q}$ is classical investment function which decreases at increasing debt d and increases at increasing output y. Putting (5) into model (1) we receive the model (6)

$$\begin{aligned}\dot{d} &= a\left(\frac{\kappa}{1+e^{md+o\rho-p\beta y}}\right)^2 - s_f\left[\beta y - (\rho + i_1 d)d\right] - \left(\frac{\kappa}{1+e^{md+o\rho-p\beta y}}\right),\\ \dot{y} &= \alpha\left\{(1-s_1)\left[(1-s_f\beta-\tau_1)y + T_0\right] + (1-s_2)(\rho+i_1 d)d + (1-s_3)\rho b\right.\\ &\quad \left. + a\left(\frac{\kappa}{1+e^{md+o\rho-p\beta y}}\right)^2 + v - y\right\}.\end{aligned} \tag{6}$$

Equilibrium $\left(d^*, y^*\right)$ of model (6) is determined by equations:

$$a\left(\frac{\kappa}{1+e^{md+o\rho-p\beta y}}\right)^2 - s_f\left[\beta y - (\rho + i_1 d)d\right] - \left(\frac{\kappa}{1+e^{md+o\rho-p\beta y}}\right)d = 0, \tag{7}$$

$$\begin{aligned}&(1-s_1)\left[(1-s_f\beta-\tau_1)y + T_0\right] + (1-s_2)(\rho+i_1 d)d + (1-s_3)\rho b\\ &+ a\left(\frac{\kappa}{1+e^{md+o\rho-p\beta y}}\right)^2 + \upsilon - y = 0.\end{aligned} \tag{8}$$

Take now the following set of economic variables and parameters:

$$\kappa = \frac{1}{4}, m = 2, o = 1, \rho = \frac{3}{100}, p = 1, \beta = \frac{1}{10}, i_1 = \frac{5}{100}, \tau_1 = \frac{4}{10}, T_0 = \frac{2}{100}, b = \frac{2}{10},$$

$$\upsilon = \frac{1}{10}, s_1 = \frac{1}{4}, s_2 = \frac{4}{10}, s_3 = \frac{7}{10}, s_f = \frac{1}{4} \text{ with free } a.$$

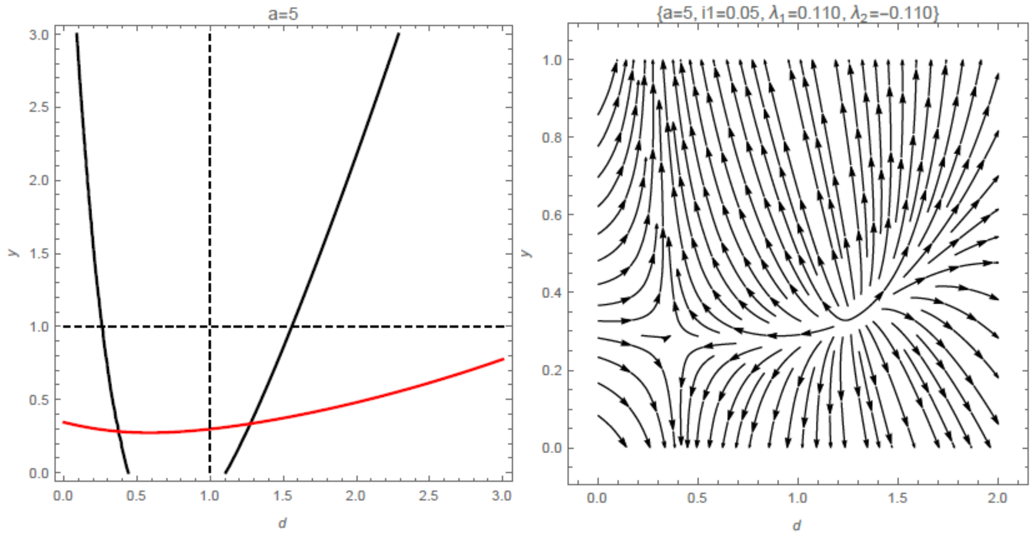
For this set of economic variables and parameters we will calculate at four increasing values of a the equilibrium $\left(d^*, y^*\right)$, the critical value of the parameter α_0 at which

$TrJ(d^*, y^*)$ is zero, $\det J(d^*, y^*)$ and the eigenvalues λ_1, λ_2 of $J(d^*, y^*)$. From economic sense the values of an equilibrium (d^*, y^*) should lie in the square $(0,1)\times(0,1)$. These results together with the graphs determining the equilibriums (the black curve is the solution of equation (7) and the red one is the solution of equation (8)) and their corresponding phase portraits are depicted in Figures 1 – 4.

In Figures 1 – 4 we can see that at increasing values of a the values of the corresponding equilibriums are getting bigger. In all these cases the eigenvalues are real and the critical values α_0 are negative. At the value $a = 25$ an equilibrium even does not exist. This situation is depicted in Figure 5.

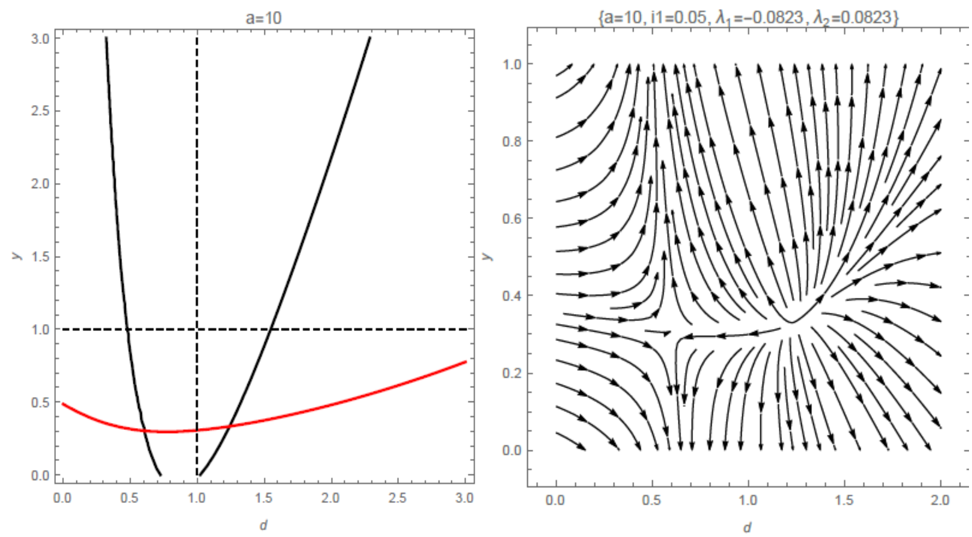
The phase portrait qualitatively changes, if for the same set of economic variables and parameters we take instead of the critical value α_0 a positive value $\alpha = 0.1$. This situation is depicted in Figure 6.

Figure 1: Existence of equilibrium for $a = 5, d^* = 0.371, y^* = 0.281, \alpha_0 = -0.212$



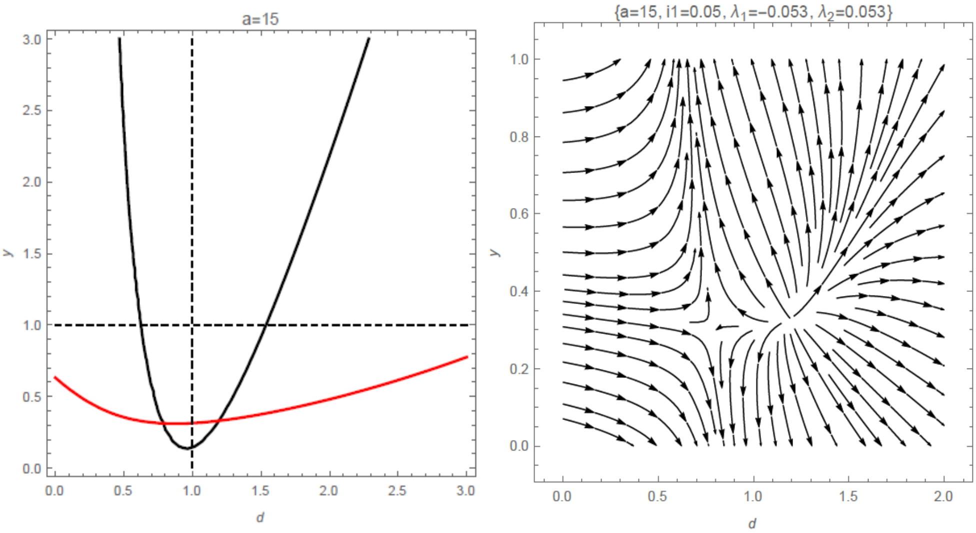
Source: the authors.

Figure 2: Existence of equilibrium for $a = 10, d^* = 0.602, y^* = 0.302, \alpha_0 = -0.148$



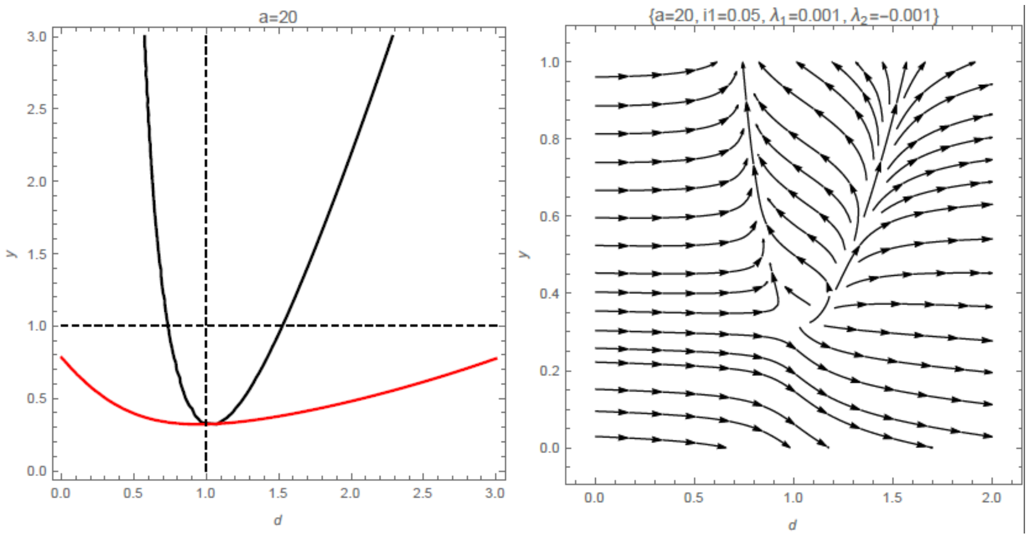
Source: the authors.

Figure 3: Existence of equilibrium for $a = 15, d^* = 0.781, y^* = 0.311, \alpha_0 = -0.094$



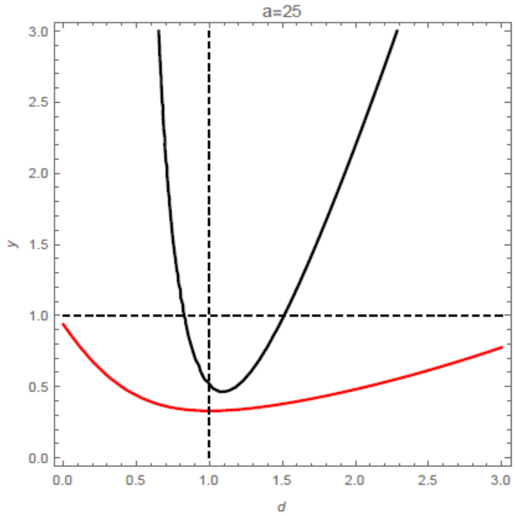
Source: the authors.

Figure 4: Existence of equilibrium for $a = 20, d^* = 0.993, y^* = 0.320, \alpha_0 = -0.016$



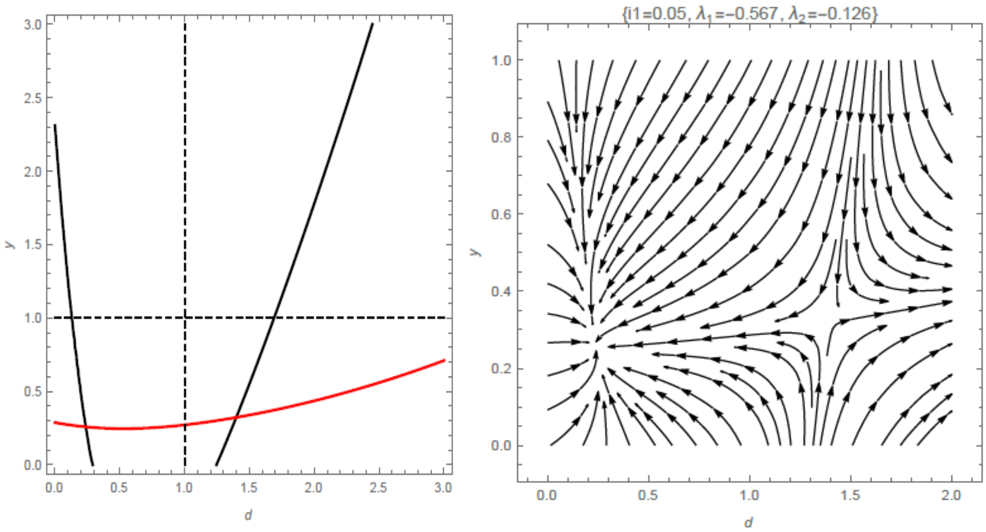
Source: the authors.

Figure 5: Non-existence of equilibrium for $a = 25$



Source: the authors.

Figure 6: $a = 3, d^* = 0.242, y^* = 0.263, \alpha = 0.1$



Source: the authors.

3. Conclusion

Asada, analyzing his two-dimensional model (1) with general forms of a function of investment $g(r,\rho,d)$ and an adjustment cost function of investment $\phi(g)$ supposed that if $\frac{\partial\phi(g)}{\partial g}$ and $\left|\frac{\partial g}{\partial d}\right|$ are sufficiently large at the equilibrium of the model then inequalities (3), which guarantee that Jacobian matrix has pure imaginary eigenvalues, will be satisfied. In this paper it is shown that for a concrete set of the functions $g(r,\rho,d)$ and $\phi(g)$ and concrete values of parameters with economic meaning it is not possible to fulfil inequalities (3).

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