PROFITABILITY OF MARRIAGE INSURANCE CONTRACTS

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Abstract

Profit testing is the process of adjusting the features of a contract (for example premiums) until the sequence of annual profits satisfies some profit criterion. Thus, it enables the incidence of profit to be taken into account in policy design. In order to evaluate cash flows and profits arising from marriage insurance, the set of 'realistic' assumptions (called the second-order basis) is taken (while the first-order-basis is the one used in premium calculation). These assumptions are related to the probabilistic structure of the marriage insurance model and the rate of interest, which for the second-order-basis are 'realistic' and for the first-order-basis are deliberately chosen. The aim of the paper is to analyze the influence of mortality dependence between spouses (modeled by copulas) on emerging costs and the profit expected to emerge for each year of the insurance period.

Key words: *emerging costs, expected profit, dependent lifetimes, multistate model, marriage insurance.*

1. Introduction

A very important issue of the actuarial analysis of the policy cash flows is the analysis of the expected profit and the expected cash flows for each year of the insurance policy. These values are the relevant elements of the set of the contract's terms, because they allow to determine the range of profit, which is taken into account in designing an insurance policy. In particular emerging costs and profit expected to emerge for each year of insured period leads to the process of adjusting the features of a contract, which is called profit testing. The paper is planned to analyze the impact of degree of dependence between the future lifetimes of married partners on actuarial values belonging to the profit testing. The new solvency regime of the European Union (Solvency II) uses worst-case scenarios for the calculation of solvency capital requirements for life insurance business. In parallel, premiums and reserves are also calculated under a set of assumptions that represent a worst-case scenario for the insurer. To our knowledge, the actuarial literature does not offer any results which may be used for construction of worst-case scenarios for valuation of multilife contracts, especially under the assumption that the insured future lifetimes are dependent.

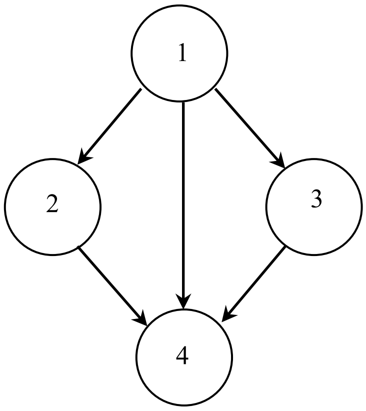
In this paper, we focus on profitability of the marriage life insurance model. In order to evaluate cash flows and profit, we make some assumptions about the rate of interest and the probability structure of the multiple state model, which describes the evaluation of insured risk, as far as its evaluation is concerned. The net premiums are calculated securely with some surplus. Then both the interest rate and the transition probabilities in the model should be carefully chosen. Moreover, the future lifetime of spouses are independent random variables and to count the distribution of their future lifetime we use Polish Life Tables. The second-order-basis is the determination of insurer's profit and loses on the basis of realistic parameters. We take into account a greater value of the interest rate than in the first-order-basis. We assume that the future lifetimes of spouses are depended random variables and the distribution of their future lifetime is modeled by copulas.

In the paper, a discrete-time model, where insurance payments are made at the ends of time intervals, will be focused on. The matrix notation will be adapted to calculations of all actuarial values analyzed in this article. In the numerical examples, we analyze the impact of the probabilistic structure of the model and the value of the interest rate on the particular elements of the profit testing of marriage life insurance.

2. Modeling Marriage Insurances

The marriage life insurance is the policy in which spouses future life is insured. The benefit is paid when one of spouses or both spouses die. We consider the case, when the benefit is paid after the death of one spouse as well as after the death of the second spouse, i.e. the Last Surviving Status (LSS). The distinguished four-states models are presented in the Figure 1.

Figure 1: A multiple state model for marriage insurance contract.



State space:
1 - both spouses are alive
2 - husband is dead
3 - wife is dead
4 - both spouses are dead

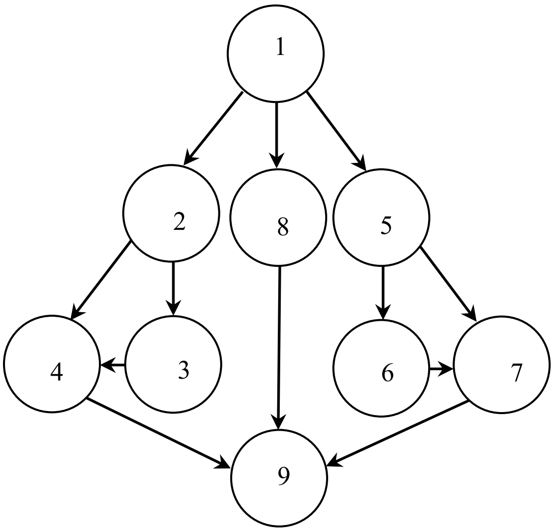
Source: the authors.

Let n is called the term of policy or the insurance period. According to LSS $n = \max\{w_x^M, w_y^M\}$, where w_x^M denotes the difference between the limit age of the man and husband's age at entry x and w_y^W denotes the difference between the limit age of the woman and wife's age at entry y.

To apply the matrix notation in case of calculating actuarial values, we modified our model to the extended multiple state model, which is presented on the graph in Figure 2. In order to achieve the modified model we used recurrent procedure described in (Dębicka, 2013).

We focus on a discrete-time model, where $X^*(k)$ denotes the state of an individual (the contract) at time k ($k \in \mathrm{T} = \{0,1,2,...,n\}$). Hence the evolution of the insured risk is described by a discrete-time stochastic process. In calculations of actuarial values of life insurances, it is usually assumed that $\{X^*(k)\}$ is modelled by a Markov chain (Christiansen, 2012; Djehjche, 2011). Under such assumption, the probability structure of our model is determined by $\mathbf{Q}^*(k) = (q_{ij}(k))_{i,j=1}^{9}$, where transition probability $q_{ij}(k) = P(X^*(k+1) = j \,|\, X^*(k) = i)$. The matrix $\mathbf{Q}^*(k) = (q_{ij}(k))_{i,j=1}^{9}$ is determined for LSS in (Dębicka et al., 2016b), in situations where future lifetimes of spouses are independent or dependent (modeled by copula).

Figure 2: An extended multiple state model for marriage insurance contract.



State space:
1 - both spouses are alive
2 - death of the husband, wife is alive
3 - husband has been dead for at least one year, wife is alive
4 - husband has been dead for at least one year, death of the wife
5 - death of the wife, husband is alive
6 - wife has been dead for at least one year, husband is alive
7 - wife has been dead for at least one year, death of the husband
8 - both spouses are dead
9 - both spouses have been dead for at least one year

Source: the authors.

The structure of interest rate determines the matrix $\mathbf{\Lambda} = \left(\lambda_{t_1 t_2}\right)_{t_1,t_2=1}^{n+1}$, where

$$\lambda_{t_1 t_2} = \begin{cases} E(\upsilon(t_2,t_1)) & for \quad t_1 > t_2, \\ 1 & for \quad t_1 = t_2, \\ E(\upsilon(t_1,t_2)) & for \quad t_1 < t_2, \end{cases}$$

and $\upsilon(t_1,t_2)$ is a discount function for $[t_1,t_2]$ period. If the interest rate is constant we have $E(\upsilon(t_1,t_2)) = \upsilon^{t_1-t_2}$ and if it is a function of time we obtain $E(\upsilon(t_1,t_2)) = e^{(t_2-t_1)R_{t_1,t_2}}$, where R_{t_1,t_2} is the short term rate.

Matrix $\mathbf{\Lambda}$ and the chain of transition matrices $\{\mathbf{Q}^*(k)\}_{k=0}^{n-1}$, under the assumption that spouses lifetimes are independent, form the first-order basis.

We will denote with $\breve{\mathbf{Q}}(k) = \left(\breve{q}_{ij}(k)\right)_{i,j=1}^{9}$, where $\breve{q}_{ij}(k) = P\left(\breve{X}(k+1) = j \middle| \breve{X}(k) = i\right)$ the realistic probability structure of the model and with $\breve{\mathbf{\Lambda}} \in \Re^{(n+1)\times(n+1)}$ the realistic structure of interest rate describes the matrix. Thus matrix $\breve{\mathbf{\Lambda}}$ and the chain of transition matrices $\{\breve{\mathbf{Q}}(k)\}_{k=0}^{n-1}$, under the assumption that spouses lifetimes are dependent, form the second-order basis.

Note that processes $\{X^*(t); t \in \mathrm{T}\}$ and $\{\breve{X}(t); t \in \mathrm{T}\}$ take values in the same space $S^* = \{1,2,...,9\}$, but their probability structures are different. Moreover, the interest rate for matrix $\breve{\mathbf{\Lambda}}$ is greater than the interest rate for matrix $\mathbf{\Lambda}$.

3. Matrix Notation

Let matrix $\mathbf{D}$ consists of probabilities of staying process $\{X^*(t); t \in T\}$ at states in each moment of insurance period

$$\mathbf{D} = \begin{pmatrix} \mathbf{P}^T(0) \\ \mathbf{P}^T(1) \\ \vdots \\ \mathbf{P}^T(n) \end{pmatrix} \in \Re^{(n+1)\times 9},$$

where $\mathbf{P}^T(k) = \left(P\left(X^*(k)=1\right), P\left(X^*(k)=2\right), \dots, P\left(X^*(k)=9\right)\right)^T$ and $\mathbf{P}^T(0) = (1,0,0,0,0,0,0,0,0)^T$ is a vector of initial distribution. Based on $\{\mathbf{Q}^*(k)\}_{k=0}^{n-1}$ we describe columns of matrix $\mathbf{D}$ as follows

$$\mathbf{P}^T(k) = \mathbf{P}^T(0)\prod_{t=0}^{k-1}\mathbf{Q}^*(t).$$

In the same way we create matrix $\breve{\mathbf{D}}$ for the second-order basis.

Let c^M (c^W) be the given lump sum paid when husband (wife) dies and c be the lump sum paid if both spouses die. Then the cash flow matrix $\mathbf{C}_{in}$, consisting of inflow to the insurer loss fund (i.e. benefits), has the following form

$$\mathbf{C}_{in} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c^M & 0 & 0 & c^W & 0 & 0 & c & 0 \\ 0 & c^M & 0 & c^W & c^W & 0 & c^M & c & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & c^M & 0 & c^W & c^W & 0 & c^M & c & 0 \end{pmatrix} \in \Re^{(n+1)\times 9}.$$

Moreover, cash flow matrix $\mathbf{C}_{out}$ consisting only of outflow from the insurer loss fund has the following form

$$\mathbf{C}_{out} = \begin{pmatrix} -p & 0 & \cdots & 0 \\ -p & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -p & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \in \Re^{(n+1)\times 9}, \tag{1}$$

where p is net period premiums paid at the first m units of the insured period ($m \le n$). Note that $\mathbf{C} = \mathbf{C}_{in} + \mathbf{C}_{out} = \left(\mathbf{C}_{\underline{in}} + \mathbf{C}_{\overline{in}}\right) + \mathbf{C}_{out}$, where $\mathbf{C}_{\overline{in}}$ and $\mathbf{C}_{\underline{in}}$ contain benefits paid immediately and due, respectively. Since all the benefits of the analyzed insurance are paid due, therefore $\mathbf{C}_{in} = \mathbf{C}_{\underline{in}}$.

Let $\mathbf{S} = (1,1,\dots,1)^T \in \Re^9$, $\mathbf{I}_k = (0,\dots,0,\underset{k}{1},0,\dots,0)^T \in \Re^{n+1}$, $\mathbf{J}_i = (0,\dots,0,\underset{i}{1},0,\dots,0)^T \in \Re^9$ for each $k = 0,1,2,\dots,n$ and $j = 1,2,\dots,9$. Furthermore, for any matrix $\mathbf{A} = \left(a_{ij}\right)_{i,j=1}^{n+1}$, let $Diag(\mathbf{A})$ be a diagonal matrix whose diagonal elements consist of a diagonal of the matrix $\mathbf{A}$.

4. Emerging Costs and Expected Profit

Let $CF_i(k)$ denotes the expected cash flows for year k if the policy holder is in state i at the start of the year. $CF_i(k)$ is the difference between insurer's incomes and outflows while $[k-1,k)$ accumulated at the moment k on condition that $\breve{X}(k+1)=i$. In other words, this is the amount of money which is expected to emerge at the end of year k under assumption that at the beginning the policy starts in state 1, i.e. $X^*(0)=\breve{X}(0)=1$. Thus for $k=1,2,3,...,n$ we have

$$\mathbf{CF}(k)=\left(CF_1(k),CF_2(k),...,CF_9(k)\right)^T.$$

We received vector $\mathbf{CF}(k)$ (under assumption that $\mathbf{C}_{in}=\mathbf{C}_{\underline{in}}$) from the following formula (cf. Dębicka, 2013)

$$\mathbf{CF}(k)=\left(-\mathbf{C}_{out}^T\,\mathbf{I}_k\mathbf{I}_k^T\breve{\mathbf{\Lambda}}-\breve{\mathbf{Q}}(k-1)\mathbf{C}_{\underline{in}}^T\right)\mathbf{I}_{k+1}. \tag{2}$$

The expected value of $\mathbf{CF}(k)$ on condition that $\breve{X}(0)=1$ is the emerging cost for year k, where $k=1,2,3,...,n$, i.e.

$$ECF(k)=\mathrm{E}\left(\mathbf{CF}(k)\middle|\breve{X}(0)=1\right)=\sum_{i\in S}CF_i(k)q_{1i}(k-1).$$

Hence $\mathbf{ECF}(k)=\mathbf{I}_k^T\cdot\breve{\mathbf{D}}\cdot\mathbf{CF}(k)$.

The matrix of the expected cash flows in the whole insurance period is a vector of expected value of $\mathbf{CF}(k)$ on condition that $\breve{X}(0)=1$, which means

$$\mathbf{ECF}=\begin{pmatrix}ECF(1)\\ ECF(2)\\ \vdots\\ ECF(n)\end{pmatrix}.$$

Note that $CF_i(k)$ could be negative, therefore the $CF_i(k)$ does not represent the expected profit to the insurer due to the fact that funds must be aside to meet the expected outgo in future years, for example for paying benefits. Accordingly, to balance the stream of cash flows in any year to be regarded as insurer's profit, the prospective reserves must be taken into account.

Let $PR_i(k)$ denote the profit expected to emerge at the end of year k per policy in state i at time $k-1$. Thus for $k=1,2,3,...,n$ we have

$$\mathbf{PR}(k)=\left(PR_1(k),PR_2(k),...,PR_9(k)\right)^T,$$

where (cf. Dębicka, 2013)

$$\mathbf{PR}(k)=\mathbf{CF}(k)+\left(\mathbf{V}^{\mathbf{T}}\,\mathbf{I}_k\mathbf{I}_k^T\breve{\mathbf{\Lambda}}-\breve{\mathbf{Q}}(k-1)\mathbf{V}^{\mathbf{T}}\right)\mathbf{I}_{k+1}, \tag{3}$$

and

$$\mathbf{V}(k)=\left(\mathbf{C}_{out}^T+\left(\sum_{t=k+1}^{n}\sum_{u=k}^{t-1}\mathbf{Q}^*(u)\mathbf{C}^T\mathbf{I}_k\mathbf{I}_k^T\right)\mathbf{\Lambda}\right)\mathbf{I}_{k+1}\in\mathfrak{R}^{n+1}.$$

Vector $\mathbf{V}(k)$ consists of net prospective reserves, which are calculated at moment $k-1$ after the payment of the benefit then due but before the receipt of the premium then due. Note that $\mathbf{V}(k)$ are calculated safety in the framework of the first-order-basis (like premiums).

The expected value of $\mathbf{PR}(k)$ on condition that $\breve{X}(0)=1$ is calculated for year k, where $k=1,2,3,...,n$, i.e.

$$EPR(k) = \mathrm{E}\left(\mathbf{PR}(k) \middle| \breve{X}(0) = 1\right) = \mathbf{I}_k^T \cdot \breve{\mathbf{D}} \cdot \mathbf{PR}(k).$$

The matrix of the expected emerging cost is a vector of expected value of $\mathbf{PR}(k)$ on condition that $\breve{X}(0) = 1$, which means that

$$\mathbf{EPR} = \begin{pmatrix} EPR(1) \\ EPR(2) \\ \vdots \\ EPR(n) \end{pmatrix}.$$

Analysis of vectors **ECF** and **EPR** is most commonly referred for considering the profitability of policies with complex designs.

5. Numerical Analysis

<u>Benefits and premiums</u>. We assume that the lump sum paid when husband or wife dies are the same and equal 1 i.e. $c^M = c^W = 1$. The lump sum paid if both spouses die is equal to 2, i.e. $c = c^M + c^W = 2$. Moreover, the constant period premium is paid when both spouses are alive.

<u>Probabilistic structure</u>. We investigate the Markov model based on the stationary Markov chain (see Wolthuis and van Hoeck, 1986; Norberg, 1989; Denuit et al., 2001). This model let us establish the joint distribution of the lifetimes of spouses, i.e. $\left(T_x^M, T_y^W\right)$, for fixed ages x and y. Heilpern (2015) derived the joint cumulative distribution function $F(w,z) = C\left(P\left(T_{x_0}^M \le w\right), P\left(T_{y_0}^W \le z\right)\right)$, where $C(u,v)$ is the copula, for $x_0 = y_0 = 60$ and the Kendall's tau coefficient of correlation $\tau = 0.076$ between random variables $T_{x_0}^M$ and $T_{y_0}^W$. These values, x_0 and y_0, are treated as a reference age for men and women in our paper. Heilpern also designated the copula $C(u,v)$, which described the dependence structure of lifetimes $T_{x_0}^M$ and $T_{y_0}^W$. It was the following Gumbel copula

$$C(u,v) = \exp\left(-\left((-\ln u)^\alpha + (-\ln v)^\alpha\right)^{\frac{1}{\alpha}}\right),$$

where $\alpha = 1.0786$. This copula was selected from the five frequently used families of copulas: Clayton, Gumbel, Frank, Ali-Mikhail-Haq (AMH) and Farlie-Gumbel-Morgenstern (FGM). It best satisfied the criterion based on the distance between the joint distribution function and theoretical distribution function induced by copula connected with Kendall's tau $\tau = 0.076$ and it was estimated for men at age $[x_0, 100)$ and women at age $[y_0, 100)$.

To the actuarial calculation we need the following survival function

$$S(w,z) = P(T_{x_0}^M > w, T_{y_0}^W > z).$$

The joint survival function $S(w,z)$ can be determined by the use of the survival copula $C^*(w,z)$ as follows

$$S(w,z) = C^*\left(S^M(w), S^W(z)\right),$$

where $S^M(w) = P\left(T_{x_0}^M > w\right)$, $S^W(z) = P\left(T_{y_0}^W > z\right)$ and (cf. Nelsen, 1999)

$$C^*(w,z) = w + z - 1 + C(1-w, 1-z).$$

Copulas are applied to calculate the elements of matrices $\breve{\mathbf{Q}}(k) = \left(\breve{q}_{ij}(k)\right)_{i,j=1}^{9}$. The probability structure of $\mathbf{Q}^*(k) = \left(q_{ij}(k)\right)_{i,j=1}^{9}$ is determined by the use of the assumption of independent future lifetime of spouses i.e. for $C^*(w,z) = w \cdot z$ and is counted based on Polish Life Tables 2011 for Lower Silesia Voivodship[1].

Interest rate. We chose the real Polish market data, related to the yield to maturity on fixed interest bonds and zero-coupon bonds from March 3, 2015[2]. We used three models of short term rate $R_{0,k}$ (k is time to maturity) to fit it to the yield curve (cf. Rezende and Ferreira, 2013). The parameters of function $R_{0,k}$ are estimated by using the least-squares method by the use of the Solver in Microsoft Excel. Svensson model of short term rate (cf. Dębicka et al., 2016a) is the most suitable for the data model.

The function R_{t_1,t_2} has the following form (cf. Dębicka and Marciniuk, 2014)

$$R_{0,k} = \beta_0 + \beta_1 \frac{\tau_1}{k}\left(1 - e^{-\frac{k}{\tau_1}}\right) + \beta_2\left(\frac{\tau_1}{k}\left(1 - e^{-\frac{k}{\tau_1}}\right) - e^{-\frac{k}{\tau_1}}\right) + \beta_3\left(\frac{\tau_2}{k}\left(1 - e^{-\frac{k}{\tau_2}}\right) - e^{-\frac{k}{\tau_2}}\right),$$

where $\beta_0 = 0.02096$, $\beta_1 = -0.01684$, $\beta_2 = 0.05844$, $\beta_3 = -0.05069$, $\tau_1 = 0.33388$, $\tau_2 = 0.57974$. Since β_0 is interpreted as a long term rate of interest, we assume that the $r = \beta_0 = 2.1\%$ and it is used to the calculation in the first-order-basis. Because r is estimated on the basis of safe bonds, we apply higher value of interest rates r_1 to the calculation in the second-order-basis. We assume that $r_1 \geq 2.1\%$.

In the first part of the numerical analysis, we would like to observe the influence of the value of interest rate for second-order basis on the expected cash flows and the profit expected to emerge. Thus we started by counting expected cash flows and expected emerging costs for year *k* in respect to particular states. We assume that husband and wife are in the same age i.e. $x = y = 60$ years old.

The net period premium payable in advance at the beginning of the time unit during the first m units ($m \leq n$) if $X^*(t) = 1$ equals (see Debicka, 2013)

$$p = \frac{\mathbf{I}_1^T \mathbf{\Lambda}^T Diag(\mathbf{C}_{in}\mathbf{D}^T)\mathbf{S}}{\mathbf{I}_1^T \mathbf{\Lambda}^T \left(\mathbf{I} - \sum_{t=m+1}^{n+1} \mathbf{I}_t \mathbf{I}_t^T\right)\mathbf{D}\mathbf{J}_1}. \tag{4}$$

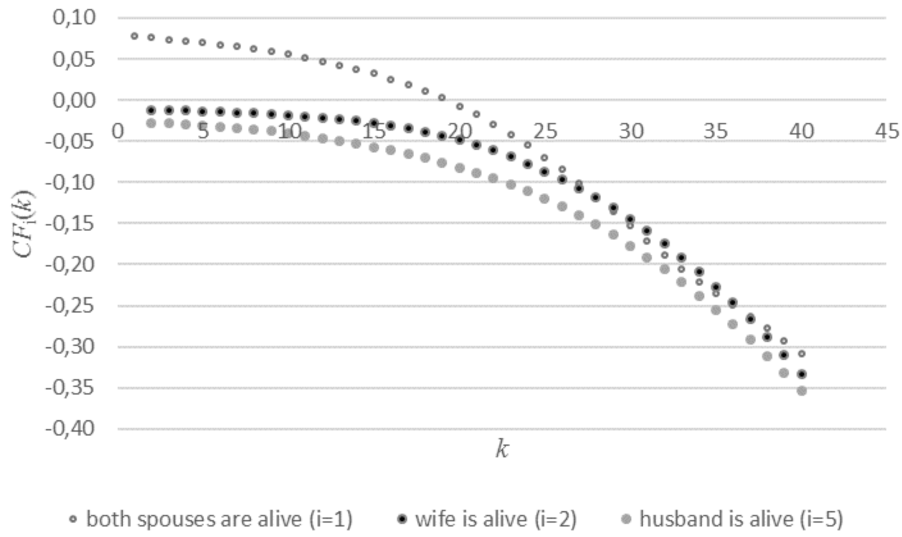
Thus the period premium (according to (4)) paid for 40 years if spouses are alive ($m = n = 40$ years) equals $p = 0.100588673$.

Note that $CF_i(k) = 0$ for the absorbing state i.e. $i = 9$ and for reflex states (that is strictly transitional and after one unit of time, the insured risk leaves this state), which has the direct transition to the absorbing state, i.e. $i = 4,7,8$. Moreover, $CF_2(k) = CF_3(k)$ because they relate to the same random event that is husband's death. Analogously $CF_5(k) = CF_6(k)$, so they are associated with wife's death. Furthermore, only the expected cash flows for state 1 depend on interest rates both for first and second-order basis; compare (2) and (1). It also means that, r_1 does not influence the expected cash flows for the rest of the state i.e. $i = \{2,3,...,9\}$. In the Figure 3 we observe the expected cash flows under assumption that $r = 0.021$ and $r_1 = 0.031$ which corresponds to the states 1, 2, 3 in the classic multistate model for marriage insurance shown in Figure 3.

[1] Polish Life Tables 2011 for Lower Silesia Voivodship are not published. The authors obtained it from Polish Main Statistical Office (Department of Wroclaw).

[2] The interest rates for the analysis were downloaded from: http://bossa.pl/notowania/stopy/rentownosc_obligacji/ (accessed March 20, 2016).

Figure 3: Expected cash flows for particular states (for $x = y = 60$, $n = m = 40$, $r = 0.021$ and $r_1 = 0.031$)



Source: the authors.

Let $CF_1^{r_1}(k)$ denote the expected cash flows for year k under assumption that interest rate for the second order basis is equal to r_1 (for $r_1 \geq 2.1\%$). It appears that based on (2), one can prove that the following relationship holds

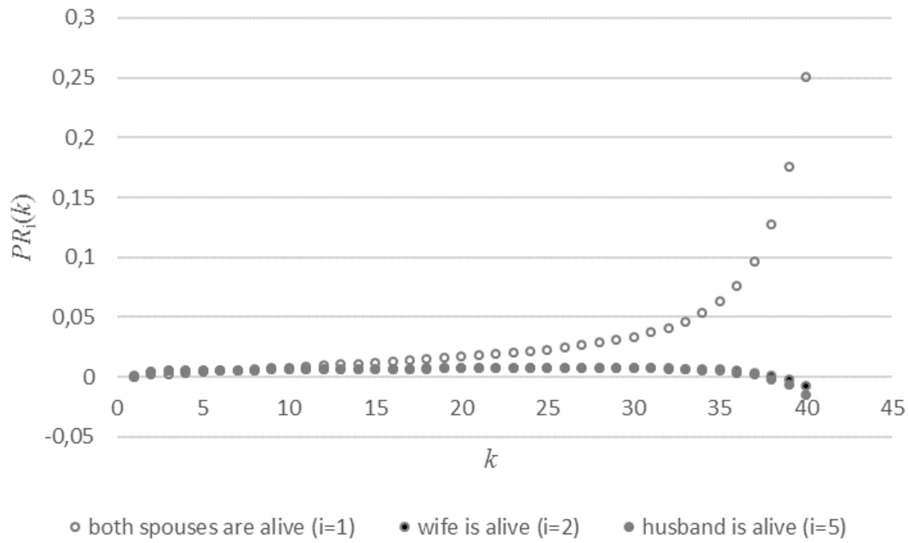
$$CF_1^{r_1}(k) - CF_1^{0.021}(k) = p \cdot (r_1 - 0.021) = 0.100588673(r_1 - 0.021),$$

for each $k = 0,1,...,n$. This means that the difference $CF_1^{r_1}(k) - CF_1^{r}(k)$ is proportional to the product of the amount of premiums and the difference between the interest rates for the second and first-order basis, i.e $r_1 - r$.

For the same reasons as for the expected cash flows, $PR_i(k) = 0$ for $i = 4,7,8,9$, $PR_2(k) = PR_3(k)$ and $PR_5(k) = PR_6(k)$. In Figure 4 we present the profit expected to emerge under assumption that $r = 0.021$ and $r_1 = 0.031$.

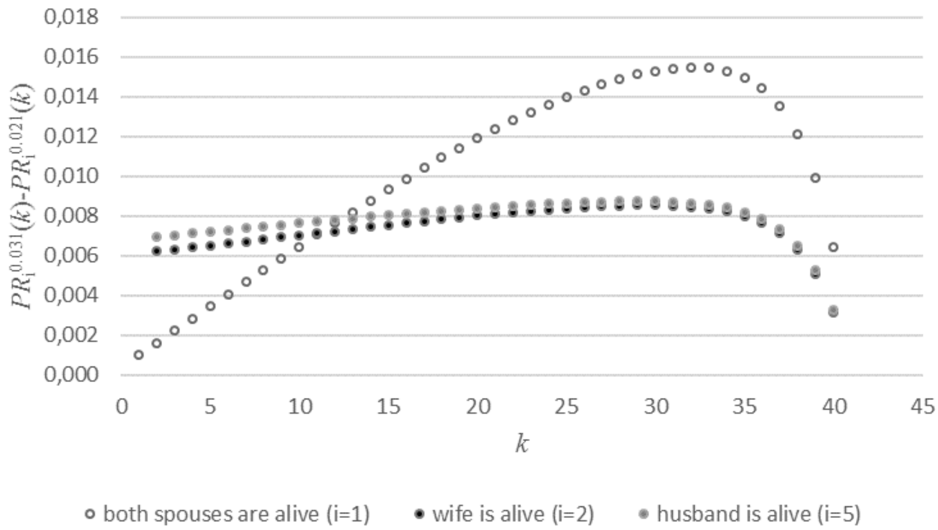
The profit expected to emerge at the end of year k at states $i = 1,2,3,5,6$ depends on both of the interest rates for first and second-order basis, compare (3). Thus in Figure 5, the difference between $PR_i^{0.031}(k)$ the profit expected to emerge at the end of year k under assumption that interest rate for the second order basis is equal to 0.031 and $PR_i^{0.021}(k)$ under assumption that interest rate for the second order basis is equal to 0.021.

Figure 4: Profit expected to emerge for particular states (for $x = y = 60$, $n = m = 40$, $r = 0.021$ and $r_1 = 0.031$)



Source: the authors.

Figure 5: Difference between profit expected to emerge for particular states (for $x = y = 60$, $n = m = 40$)

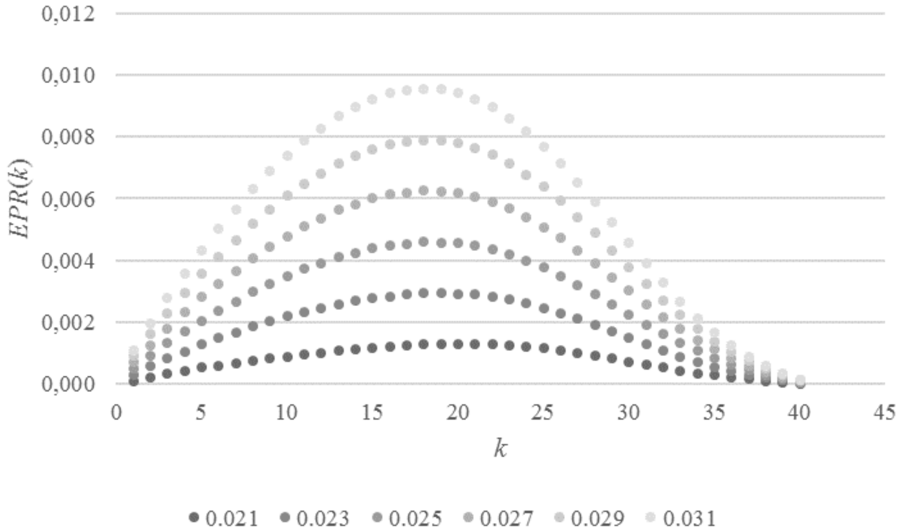


Source: the authors.

It is obvious that the greater the difference between r_1 and 0.021, the bigger $PR_i^{r_1}(k) - PR_i^{0.021}(k)$ growth. Note that the value of the difference between profit expected to emerge growth is similar to the state indicating the death of one of the spouses. It is easy visible, that $PR_i^{0.031}(k) - PR_i^{0.021}(k)$ for the death of the wife is a little bigger than in case of the death of her husband. This is directly related to the difference between the duration of the lifetime of husband and wife.

In Figure 6 the expected profit to emerge in the whole marriage life insurance period depending on the value of interest rate for the second-order basis is presented.

Figure 6: Expected profit to emerge in the whole marriage life insurance period (**EPR** for $x = y = 60$, $n = m = 40$, $r = 0.021$ and $r_1 = 0.021, 0.023, ..., 0.031$)



Source: the authors.

Note that, the greater the difference between interest rates of the first and second-order basis, the bigger the expected profit to emerge. A big impact on the value of $EPR(k)$ has the prospective reserve (compare (3)), which is visible in the shape of graphs in Figure 6.

In the second part of the numerical analysis, we would like to observe the influence of probabilistic structure for the second-order basis on the expected cash flows **ECF** and the expected cash flows **EPR** in the whole insurance period depending on age at entry of the spouses. We would like to investigate only the impact of the probability structure, so we assume that $r_1 = r = 0.021$. We made calculations for husband and wife at age 60 or 65 i.e. $x, y \in \{60, 65\}$.

The period premium p (counted according to formula (4)), the insurance period n and the period of paying premiums m for particular pairs of spouses under assumption that $r = 0.021$ are presented in Table 1.

Table 1: Period premiums for marriage insurance contracts

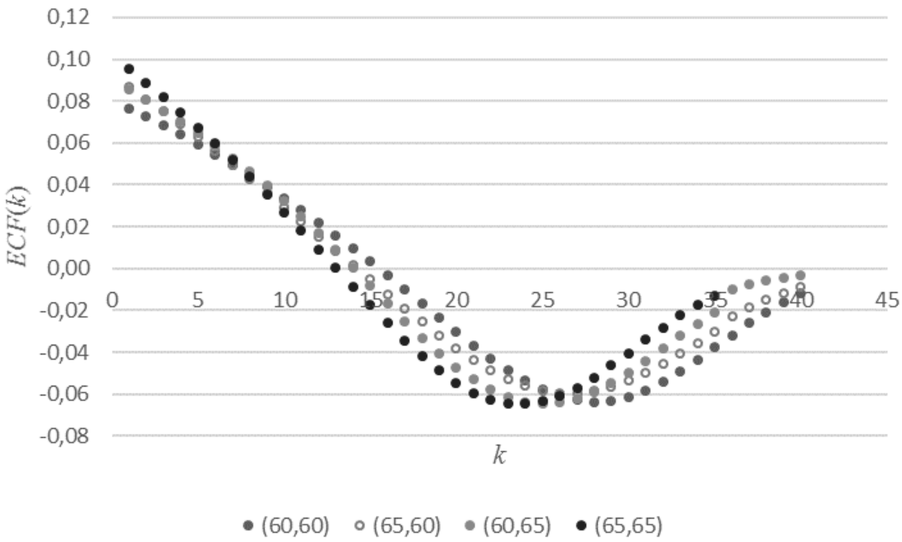
(x, y)	(60, 60)	(65, 60)	(60, 65)	(65, 65)
n	40	40	40	35
m	40	35	35	35
p	0.100588673	0.116768433	0.112917333	0.128969263

Source: the authors.

The period premium of marriage insurance contract increases with age of the spouses. Note that for a younger husband and an older wife the premium is lower than for an older husband and a younger wife. One can observe that, the man's age has a greater impact on the premium.

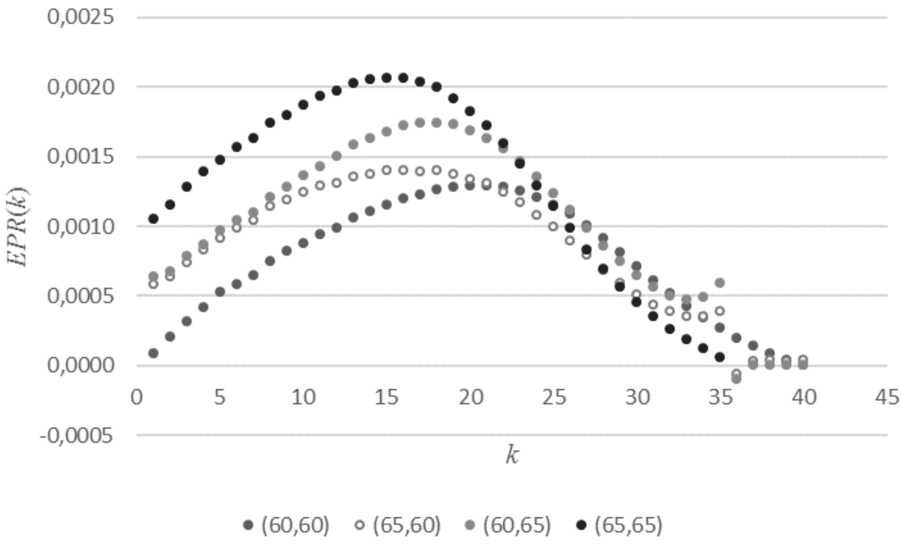
In Figure 7 and Figure 8, expected cash flows and expected profit to emerge in the whole marriage life insurance period depending on spouses age at entry are presented.

Figure 7: Expected cash flows in the whole marriage life insurance period (**ECF** for $(x, y): x, y \in \{60, 65\},\ r_1 = r = 0.021$)



Source: the authors.

Figure 8: Expected profit to emerge in the whole marriage life insurance period (**EPR** for $(x, y): x, y \in \{60, 65\},\ r_1 = r = 0.021$)



Source: the authors.

Observing graphs in Figure 8, it can be seen that in case of peers expected profit is greater in the first half of the insurance period for the elderly spouses, later the situation is changing. Moreover, in case of married couples where the husband is younger, expected profit is higher than for couples in which the man is older.

All calculations are made with the use of the authors' own programs written in C++.

6. Conclusion

In the paper (cf. Dębicka et al., 2016b) we analyze the actuarial values of the marriage insurance (premiums, benefits, technical reserves). This article is the continuation and final step of the analysis of the cash flows of the marriage insurance. An important element of designing the policy cash flows is the analysis of the expected profit and the expected cash flows for each year of the insurance policy. It allows to determine the range of profit. The original contribution is to determine the profits for multilife insurance product under realistic assumptions assumed for the first and the second-order basis (independent and dependent probabilistic structure, respectively). For this purpose we determine the transition probability matrix, which in this case is defined by using Polish Life Tables and the copula for real Polish data. Moreover, we take into account the real interest rate, determined on the basis of data from the Polish market. Additionally, we have also shown the applicability of matrix notation to calculate the required actuarial value, which simplifies the analysis and calculations. The original contribution are also programs written in C++.

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