COMPARISON OF APPROACHES FOR ASSET PRE-SELECTION IN PORTFOLIO TRACKING

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Abstract

In passive portfolio management, two main approaches are used in pre-selecting assets out of which a tracking portfolio should be later created: the principle of simple market capitalization and the principle of industry-capitalization stratification. With the intent to demonstrate the general (but not possibly universal) superiority of the market capitalization criterion, the paper investigates how the two market capitalization approaches perform in real case studies of portfolio tracking and compares them with the approach of random diversification. Two aspects that are investigated therewith are the effect of the number of assets on tracking performance and the role of transaction costs, both fixed and variable. Through scenarios distinguishing small, moderate and large amounts of investment as well as transaction costs, it is shown that simple market capitalization is superior for asset pre-selection under almost every setting considered and can be recommended as reliable.

Key words: *asset pre-selection, market capitalization, random sampling, transaction costs, quadratic tracking.*

1. Introduction

Each method of portfolio selection requires that a pre-selection of assets has been made and only the pre-selected assets are given a chance for fund allocation in the investment process. It is both unfeasible and uneconomical to consider all the assets that are traded in a market, and some criterion guiding the asset pre-selection must be put to work. Several methods are thinkable and they usually hinge on some economic characteristics of assets under advisement such as their share on the market (i.e. their market capitalization), their industrial affiliation, their past performance and especially on a number of fundamentals (that reflect the economics of enterprises in question and that are extracted from financial information in balance sheets, income statements and commixed with other non-financial information). In contrast, from a purely statistical point of view, it is appealing to employ a criterion of randomness in the process of asset pre-selection which stems from the predominant role of random sampling in statistical analyses. Never the less, it must be admitted that the technique of random sampling in portfolio selection is frequently utilized in explaining the benefits of diversification (see e.g. Elton et al., 2009, pp. 59-60). Juxtaposing two methods of asset pre-selection, the paper compares the most frequent criterion of market capitalization with the method of random sampling, having the resolve to gain some insight and perhaps formulate useful investment advice for investors. Both methods are considered both in a simple version (when the industrial categorization is not considered) and a stratified variant (with the industrial categorization fully reflected) and evaluated in various settings with respect to several factors that influence the outcome of the investment process: the number of assets that result from the pre-selection, the relationship between the amount invested and the size of transaction costs. Set to the environment of the US equity market, 20, 30, 40, 50, 75 and 100 shares are sequentially pre-selected for portfolio optimization based on quadratic index tracking and a total of five investment scenarios are considered and compared that are tagged here for practical purposes as "SM" (small sum invested – moderate transaction costs), "MM" (moderate sum invested – moderate transaction costs), "LM" (large sum invested – moderate transaction costs), "MS" (moderate sum invested – small/no variable transaction costs) and "ML" (moderate sum invested – large variable transaction costs). By comparing the outcome for the trio of the scenarios SM, MM and LM and the trio of the scenarios MS, MM and ML, it is possible to see how the investment process may benefit from economies of scale since with high amounts of investment transaction costs become negligible and their influence on the investment outcome disappears. The choice of quadratic tracking complies with the favour that is conferred on this approach to tracking portfolio selection in the literature (see e.g. Rudolf et al., 1999, p. 86).

To be more explicit about the goal of the paper, the paper aims to investigate how both the simple and industry-stratified market capitalization approaches perform in real case studies of portfolio tracking and to compare them with the approaches of simple and industry-stratified random diversification. In this comparison, two other factors are given extra consideration, i.e. the effect of the number of asset pre-selected and the size of transaction costs relative to the invested sum. It is ascertained that the simple market capitalization criterion of asset pre-selection is uniformly advisable in comparison to the other three asset pre-selection methods.

The remainder of the paper consists of 4 more sections. Section 2 gives an overview of asset pre-selection approaches emphasized in the academic literature and then focuses upon the role of transaction costs in portfolio selection. Section 3 is preparatory providing a needful exposition for the analysis that ensues and is presented in Section 4. Finally, the last section concludes.

2. Asset Pre-selection and Transaction Costs

In recent years, there has been a growing interest in objective methods of assessing investment returns. Attention is shifting in this from absolute returns to relative returns because they embody directly the aspect of comparison since they are constructed as absolute returns minus returns of a benchmark (e.g. a hedge fund or a market index). Because performance evaluation based on the comparison with a well-defined benchmark is possibly more complex and more informative than performance evaluation per se without any connection drawn to an existing model of performance. This is one of the reasons why index tracking is a popular portfolio selection technique that rests in constructing such a portfolio that imitates either through its returns or values the returns or values of a suitably chosen market index. The most common technique of index tracking is based on minimizing the sum of square deviations between the tracking portfolio returns and the benchmark returns. This sum is then used in portfolio optimization as the objective function, and is frequently and imprecisely referred to as tracking error (albeit technically it is the deviation between the tracking portfolio returns and the benchmark returns that expresses tracking error as such).

The process of tracking portfolio selection begins with asset pre-selection and is gravely influenced by the benchmark and what it represents. If the benchmark is a market index (hence the name of index tracking), the pre-selection usually relates only assets that make part of the index and are thus its constituents. This introductory stage of portfolio selection may and does have a considerable impact upon the resultant composition of the tracking portfolio and subsequently upon its performance. In empirical and academic studies various techniques or criteria of asset pre-selection are presented and they may be roughly broken down into two broad categories: simple replication methods and stratification methods.

Replication methods attempt to mimic the composition and structure of the benchmark. A natural method here is simple replication, which is the easiest method and a method quite reasonable (see Prigent, 2007, p. 105). The tracking portfolio under this method is constructed by investing in the constituent assets of the index. If the investment is made into all assets in the index proportional to their shares that they have in the index, then it is spoken of full replication. The assets may, however, be ranked according to their shares in the index. Only top few assets may be pre-selected and the weights of these pre-selected assets in the portfolio are again derived from their shares in the index. This modification is called partial replication and in fact corresponds to pre-selecting assets on the basis of simple market capitalization. That the criterion of market capitalization applies in this case follows from the fact that the weights of assets in an index are derived from the market capitalizations of its constituents. The simple replication methodology in both its basic variants, however, associates itself with high transaction costs because investing into each (or possibly almost each) asset of the index may be expensive and not tenable if the index is modified frequently. In such a case, with the changed composition of the portfolio, it is necessary to change the entire replication portfolio in order to follow a new definition of weights in the portfolio. In this regard, e.g. Foccardi and Fabozzi (2004) implemented an optimal investment strategy with time series clustering that helps build parsimonious tracking portfolios under full replication of an index.

Frequently, several criteria are implemented in asset pre-selection, and a good example for this is the study Rey and Seiler (2001) who utilized three financial criteria, viz. market capitalization, turnover and relative turnover, in order to investigate the impact of different pre-selection choices varying according to these three criteria as well as the number of assets upon tracking error. They showed that tracking error decreases with increasing the number of assets, though not in a systematic way, and that including more and more assets does hardly result in significant lower tracking errors. They conclude that the trade-off between the (higher) number of assets and the resulting (lower) tracking error that might be anticipated disappears and becomes unfavourable if transaction costs are taken into account. Furthermore, they found the superiority of the market capitalization criterion that produced on average the lowest tracking error (see Rey and Seiler, 2001, pp. 23-24). Also Alexander and Dimitriu (2005) pre-selected assets on the basis of market capitalization and inquired into the effect of the size of the tracking portfolio upon its performance. They showed that portfolios with less than 20 assets deliver lower performance than the index and recommended more populated portfolios since they tended to outperform the benchmark index (see Alexander and Dimitriu, 2005).

In contrast to replication methods, stratification methods suppose that assets are classified into several disjunctive categories usually reflecting industrial characteristic or country affiliation of issuers (see Rudd, 1980, p. 60). After having assets stratified according to categories (industries, countries, and the like), the criterion of market capitalization qualifies assets for pre-selection into the tracking portfolio. The approach of stratified market capitalization was employed e.g. by Larsen and Resnick (1998) who found that tracking portfolios composed of large-capitalization assets are preferable to those made up of small-capitalization assets in terms of tracking error. Moreover, they found stratification across industry groups contributive only to portfolios created of large-capitalization assets (see Larsen and Resnick, 1998, p. 59). The method of stratified market capitalization was also utilized by Focardi and Fabozzi (2004) and Frino et al. (2004) with conforming results.

Transaction costs encompass trading costs both explicit (direct) and implicit (indirect). The former group of transaction costs represents those that are disclosed prior to the trade and includes commissions, markups, and other fees. Representing the costs that are not determined until after the execution of a trade or set of trades is completed, implicit costs stem from the bid-ask spread. Assets (or rather shares) issued by large companies with high market capitalization are mostly liquid and they have thus narrow bid-ask spreads. This is but not true for small companies whose assets (in particular shares) tend to be less liquid and have higher bid-ask spreads. Trading such assets is at the cost of increased trading costs that present themselves as illiquidity costs following from wide bid-ask spreads. Several studies examined the influence of transactions costs upon performance, out of which four with their findings are summarized in the following text.

In their two types of index tracking models based on mixed-integer linear programming, Mezali and Beasley (2014) recognized both fixed and variable transaction costs arising at portfolio construction as well as with portfolio rebalancing. Their optimization tasks included constraints on the number of assets as well as the limits on total transaction costs. Gustaroba and Speranza (2012) formulated a similar model covering both fixed and variable costs with restrictions on the number of assets in the tracking portfolio. In comparison to the available budget of 100,000 for the investment, fixed costs were set to 12 and variable transaction costs incurred with purchasing or selling assets were calculated proportional to asset values as 1 % of the value of traded assets. Whilst investigating properties of the cointegration-based portfolio tracking approach, Alexander and Dimitriu (2004), recognized in line with some earlier cited studies both explicit brokerage fees in a fixed amount of 20 basis points on each trade value and implicit trading costs derived from the bid-ask spread. For these, the repo costs, assumed normally small, were computed conservatively at 0.25 % of the increase in the short position in a bull market for a particular asset and at 0.35 % on the same amount in a bear market. Furthermore, Domowitz et al. (2001) analyzed the size and determinants of transaction costs in 42 countries between September 1996 and December 1998. Their results point to the existence of substantial differences amongst countries, especially between emerging markets and developed markets. Explicit costs were measured as a percentage from the difference between the trade price and a benchmark price on the day of the trade. For large capitalization portfolios in developed markets, management fees were 0.25 % and trading costs were set to 0.4 % of the traded amount. For such portfolios in emerging markets, management fees were set to 1 % and additional trading costs were calculated as 1.25 % of the traded amount.

3. Model of Portfolio Tracking

This section explains how tracking portfolios are constructed and how transaction costs are implemented. Only the quadratic tracking is paid attention in the paper since it is a common and, so-to-speak, prevalent method applied in portfolio tracking. Amongst the cited literature it is encountered or described in Alexander and Dimitriu (2004, 2005), Larsen and Resnick (1998), Prigent (2007, p. 104), Rey and Seiler (2001) and Rudolf et al. (1999).

The idea of a tracking approach to portfolio optimization is to minimize the discrepancy that comes from the difference between tracking portfolio returns and benchmark returns. This difference is termed as tracking error and the discrepancy is interpreted in quadratic tracking as a sum of tracking error squares (in fact this discrepancy is tracking error variance). The portfolio weights are determined with the aid of a set of historical returns of pre-selected assets and of a benchmark and are further utilized in a prospective fashion – though determined from past data they are employed for future investing. The optimization model does not take into account transaction costs that are incurred with a particular solution. Transaction costs are treated in the optimization model as exogenous and apply once the optimal solution for weights has been found. Both fixed and variable transaction costs of trading are distinguished here, in which variable transaction costs come in a total of three forms.

Assume that a history of T historical observations of logarithmic returns is available and that the tracking portfolio is to be composed of k assets. Let $\mathbf{Y} = (Y_1, ..., Y_T)'$ denote a $(T \times 1)$ vector of benchmark returns, and $\mathbf{X} = (\mathbf{x}_1 | ... | \mathbf{x}_T)'$ denote a $(T \times k)$ matrix of returns of the k assets that are to be represented in the tracking portfolio. In the ensuing exposition, elements of vectors are denoted by left-hand subscripts in conjunction with curly brackets, e.g. an i-th element of a vector $\mathbf{z}$ is written as $\{\mathbf{z}\}_i$. The symbol $\boldsymbol{\omega}$ stands for a $(k \times 1)$ vector of unknown portfolio weights that are obtained by minimizing the following quadratic optimization problem

$$\min_{\boldsymbol{\omega} \in \Re^k} (\mathbf{Y} - \mathbf{X}\boldsymbol{\omega})'(\mathbf{Y} - \mathbf{X}\boldsymbol{\omega}) \quad \text{subject to} \quad \boldsymbol{\omega}'\mathbf{1} = 1 \quad \text{and} \quad \text{possibly some other constraints,} \tag{1}$$

in which $\mathbf{1}$ is an $(k \times 1)$ vector of ones. This general formulation of the optimization task allows an extension and can be complemented by the constraint banning short sales requiring that $\{\boldsymbol{\omega}\}_i \geq 0$ for $\forall i \in \{1,...,k\}$. This constraint is utilized in the practical investigation and long-only positions are sought. In addition, another constraint might be the restriction that average tracking portfolio returns should be above average benchmark returns, written in formal terms as $T^{-1}\sum_t Y_t < T^{-1}\sum_t \boldsymbol{\omega}'\mathbf{x}_t$. The left-hand expression is the average benchmark return over the sample of T historical observations, whereas the expression occupying the right-hand side is the corresponding average of tracking portfolio returns $\boldsymbol{\omega}'\mathbf{x}_1, \ldots, \boldsymbol{\omega}'\mathbf{x}_T$.

Suppose that a budget B is available for the investment and the investor faces fixed transaction costs φ_F and three varieties of transaction costs. Variable transaction costs are charged to the number of assets traded and the number of holdings acquired or sold (although only long positions are considered here) and they also are applied as a proportion to the sum invested as well. The lump charges of variable costs are denoted as χ_A per asset traded and χ_H per holding of an asset purchased or sold. The percentage of variable costs that lowers the sum invested is denoted by χ_S. Suppose that there are $k^{\#}$ non-zero weights in the optimal solution $\boldsymbol{\omega}^{\#}$ to (1) extended by the requirement of short sales exclusion and assume further that the prices of assets at the instance of portfolio constructions are organized in a $(k \times 1)$ vector $\mathbf{P}$. In consequence, the amounts of transaction costs applicable to the budget B are: φ_F (fixed costs), $k^{\#} \cdot \chi_A$ (variable costs applied to the number of assets traded), $\chi_H \cdot \sum |B \cdot \{\boldsymbol{\omega}^{\#}\}_i / \{\mathbf{P}\}_i|$ (variable costs applied to the number of holdings) and $B \cdot \chi_S$ (variable costs charged to the sum of the investment).

Since the investor desires to exhaust the entire budget that he has available and the variable costs are also dependent on the number of holdings, the holdings acquired for an i-th asset are not given by $B \cdot \{\boldsymbol{\omega}^{\#}\}_i / \{\mathbf{P}\}_i$ (of course where $i \in \{1,...,k\}$) but are derived from the actual sum that can be allocated to the investment, say, $B^{\S}$. This sum is obtained by solving

$$B^{\S} = B - \varphi_F - k^{\#} \cdot \chi_A - \chi_H \cdot \sum |B^{\S} \cdot \{\boldsymbol{\omega}^{\#}\}_i / \{\mathbf{P}\}_i| - B \cdot \chi_S . \tag{2}$$

The appearance of divers B's in (2) follows from the fact that the percentage charge χ_S is applied to the original sum invested whilst the unit chare χ_H per holding is related to the sum which is then available for allocation. This equation stipulates that the actual disposable sum is $B^{\S} = [(1 - \chi_S) \cdot B - \varphi_F - k^{\#} \cdot \chi_A] / (1 + \chi_H \cdot \sum |\{\boldsymbol{\omega}^{\#}\}_i / \{\mathbf{P}\}_i|)$ and the acquired number of holdings for an i-th asset are therefore $B^{\S} \cdot \{\boldsymbol{\omega}^{\#}\}_i / \{\mathbf{P}\}_i$, $i \in \{1,...,k\}$. No rounding of holdings is made and also non-integer volumes of trades are possible.

The next section instructs how this framework is implemented, depicts the settings of the practical investigation and also presents the results achieved.

4. Practical Investigation and Results

In the practical investigation a total of five scenarios are compared that are ancillary in assessing the effect of the number of pre-selected assets and isolating the role of transaction cost upon the performance of the resultant tracking portfolio. The five scenarios differ in the amounts of investment and transaction costs that are associated with trading and three categories of investment sums as well as of transaction costs are distinguished: small (S), moderate (M) and large (L). The five scenarios are labelled here as the acronyms "SM", "MM", "LM", "MS" and "ML", in which the initial letter represents the largeness of investment sum and the second letter stands for the magnitude of transaction costs. For instance, the scenario MS pertains to the situation when a moderate budget is meted out for investing and small fees are charged at trading. The settings of these five scenarios are declared in Table 1. For clarity of presentation, the scenario MM is reported twice. The scenarios SM, MM, and LM are to be compared mutually and the same logic passes on to the scenarios MS, MM and ML.

Table 1: Settings of the five investment scenarios considered

Parameter	Medium transaction costs scenarios			Medium investment sum scenarios		
	SM	MM	LM	MS	MM	ML
B	\$1,000	\$10,000	\$100,000	\$10,000	\$10,000	\$10,000
φ_F	\$10	\$10	\$10	\$10	\$10	\$10
χ_A	\$1	\$1	\$1	\$0	\$1	\$3
χ_H	\$0.01	\$0.01	\$0.01	\$0	\$0.01	\$0.03
χ_S	0.001	0.001	0.001	0	0.001	0.003

Source: the authors.

All these five scenarios were used in tracking the S&P 500 Index but with four different methods of asset pre-selection. From amongst the 504 constituents of the S&P 500 Index (as of December 2014), 20, 30, 40, 50, 75 and 100 equities were successively pre-selected to qualify for the optimization task displayed at (1). (Of course, the actual valid number of constituents was somewhat lower owing to the need to have a sufficiently long history of past logarithmic returns. Some of the constituents began to record their history after the start of the in-sample period considered in the paper.) The four different methods of pre-selection were: simple sampling, simple market capitalization, stratified sampling and stratified market capitalization. The two sampling methods consisted just in making a random draw from the universe of the S&P 500 Index equities, either simple or stratified where the 10 Global Industry Classification Standard (GICS) sectors were respected. The list of these sectorial categories is as follows: Consumer Discretionary (85 equities), Consumer Staples (38 equities), Energy (41 equities), Financials (89 equities), Health Care (56 equities), Industrials (67 equities), Information Technology (67 equities), Materials (27 equities), Telecommunications Services (5 equities), Utilities (29 equities). The market capitalization methods made a pre-selection of equities for optimization on the basis of their market capitalization, again either without recognizing their sectorial affiliation and then stratifying across the 10 GICS sectors. The equities were ranked in descending order after their market capitalization and then a certain number of top equities were assigned for the optimization.

With the stratified sampling and market capitalization methods some rounding errors had to be taken into account in some cases as it was not possible to keep to the exact structure of equities across the sectors, but this has no relevance upon the findings. The details on the rounding imprecision that arises with the stratified methods of pre-selection for the 10 GICS sectors are the content of Table 2. This table displays the sectoral composition of the S&P 500 Index in comparison to the actually considered numbers of pre-selected assets in individual scenarios for the six sample sizes, 20, 30, 40, 50, 75 and 100 assets. For example, in pre-selections of 75 assets the Consumer Discretionary sector was represented by 13 equities (which is implied by the calculation $85 / 504 \times 75 \doteq 12.649$) that were chosen at random under stratified sampling and as the 13 most capitalized enterprises in this sector.

Table 2: Numbers of equities selected across the GICS sectors with the stratification methods

Sector	Number of assets	Pre-selected number of assets					
		20	30	40	50	75	100
Consumer Discretionary	85	3	5	7	8	13	17
Consumer Staples	38	2	2	3	4	6	8
Energy	41	2	2	3	4	6	8
Financials	89	3	5	7	9	13	18
Health Care	56	2	3	4	6	8	11
Industrials	67	3	4	6	7	10	13
Information Technology	67	3	4	5	6	10	13
Materials	27	1	2	2	3	4	5
Telecommunications Services	5	0	1	1	1	1	1
Utilities	29	1	2	2	3	4	6
Total	504	20	30	40	50	75	100

Source: the authors.

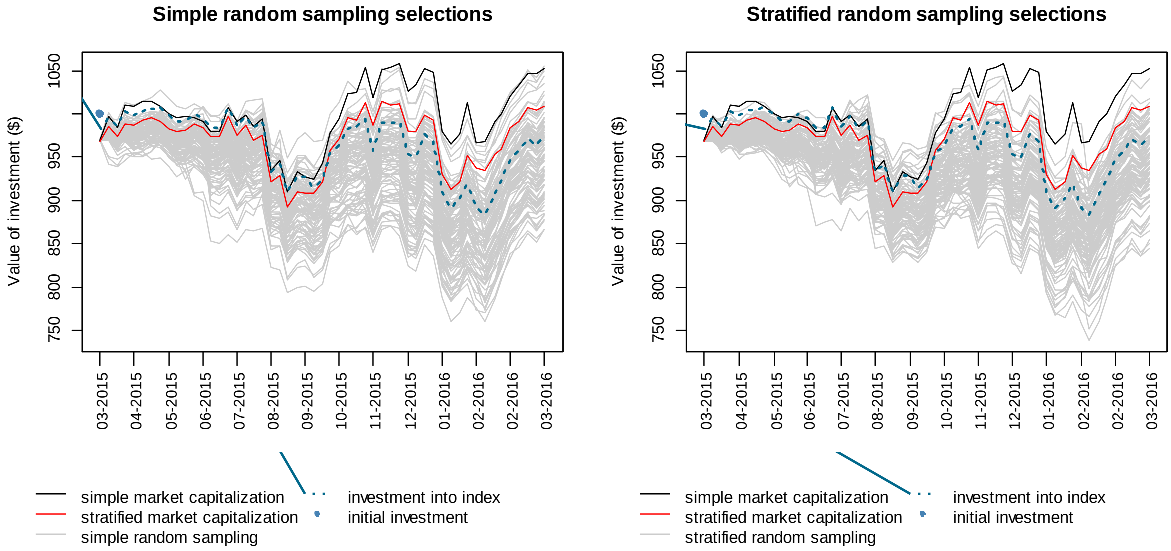
Weekly data of historical logarithmic returns were made use of in the investigation and they were divided into two sub-periods: the in-sample period served the purpose of tracking portfolio selection and the out-of-sample period represented the ex post investment period for the subsequent performance evaluation. The in-sample-period ran from 9 January 2012 to 30 March 2015 (counting thus 169 effective weekly returns) and the out-of-sample period stretched over a period one year long from 30 March 2015 to 28 March 2016 (representing 53 effective weekly returns and prices).

On the basis of the observations in the in-sample period, the weights of long-only tracking portfolios were determined using (1) for each scenario and for 20, 30, 40, 50, 75 and 100 equities pre-selected with one of the four methods: simple sampling, simple market capitalization, stratified sampling and stratified market capitalization. In contrast to the sampling methods, the pre-selection conducted by means of either market capitalization method is for each number of equities deterministic and unique. There is only one possibility to select 20, 30, 40, 50, 75 and 100 constituents of the S&P 500 Index so that their market capitalization tops the ranked list. For each random draw the pre-selection result of simple sampling and stratified sampling from the basket of S&P 500 Index components should be unique (until all choices of sampling are exhausted and except certain repeated draws). With this in mind, for each scenario and for each number of pre-selected equities under consideration a total of 100 independent draws (naturally without replacement) were performed and the performance results were averaged or summarized in a conveniently informative way.

An example of a typical situation that results is shown in Figure 1 and this one is depicted for the scenario SM when 20 equities are pre-selected for tracking the S&P 500 Index. To some extent the information is doubled for convenience. In both graphs the light-blue dot about \$1000 represents the initial investment, but the values of the constructed tracking portfolios as well as the investment to the index begin at a somewhat lower value. This is due to the fact that this initial investment of \$1000 is subtracted of the trading costs in line with the description given about relationship (2) and according to the settings of the scenario SM declared in Table 1. The portfolios were created as of the last day of the in-sample period, i.e. 30 March 2015, based on the closing equity prices reported at this day (which is a slight simplification as these prices were used to calculate the last return of the in-sample period as well as to determine the holdings of the tracking portfolios). Keeping the holdings fixed, their values were monitored over the out-of-sample period and is depicted in the graphs. The same procedure was utilized for the index, into which a similar investment of 1000 was made; this initial sum was reduced by the corresponding trading costs and the holdings were computed. Both graphs then display how the values of the tracking portfolios arising from the two market capitalization methods of pre-selection as well as the investment into the index evolved over the out-of-sample period. The respective trajectories are shown by solid black lines (simple market capitalization), by solid red lines (stratified market capitalization) and by dashed blue lines (the investment into the index). The graph in the left-hand part of Figure 1 exhibits in addition the values of the tracking portfolios constructed from equities pre-selected by means of simple random sampling, whereas the graph in the right-hand part does this for the tracking portfolios for which equities were pre-selected by stratified random sampling. Each such portfolio is shown by solid gray lines and corresponds to one particular draw of a hundred. The reports for simple and stratified random sampling pre-selections are separated to make them useful. There are three striking features about these two graphs. First, the constructed tracking portfolios as well as the investment into the index tend to follow the same pattern of development. Second, the tracking portfolios emerged from pre-selections by the simple and stratified market capitalization methods dominate the investment into the index and outperform it. Third, the best performance, at least in the second half of the out-of-sample period, can be attributed in terms of portfolio values to the tracking portfolio arising from the simple market capitalization and the worst performance on average (and in the majority of the draws) to the tracking portfolios emerging from both the simple and stratified random sampling. Of course, no universal statement may be issued as this is only the situation for 20 assets pre-selected under the scenario SM. The whole picture is thus compressed in Tables 3 and 4.

Table 3 organizes the performance results for the three moderate investment sum scenarios MS, MM and ML and Table 4 reports these results for the moderate transaction costs scenarios SM, MM and ML. In the tables, simple market capitalization is reported as "simMC", stratified market capitalization as "strMC", simple sampling is shown as "simSam" and stratified sampling displayed under "strSam". The tables show for each situation several characteristics that are instrumental in assessing the properties of the asset pre-selection methods considered and rank them in respect of their superiority, if possible. The mean return and the information ratio are on a weekly basis (per septimanam) and embody the rate of return and the performance of the tracking portfolios in relation to the rate of return and the performance of the underlying S&P 500 Index. In other words, the mean return is the mean active return and the information ratio is the return-to-volatility ratio that is derived from active returns. A positive mean active returns points to a better rate of return yielded by the tracking portfolio than by the benchmark index, whereas a positive information ratio points to a better performance for the tracking performance than for the index. Both these indicators are generally desired maximum. For the two sampling methods of pre-selection the average values are understandably reported. There are also three questions: "Is better than simMC?", "Is better than strMC?", "Is better than the index?". These three questions are answered for the market capitalization methods in a binary yes/no way, but for the sampling methods the percentages of cases are reported in which the tracking portfolios fare better than those for which the pre-selection was done by the simple market capitalization method (simMC) and by the stratified market capitalization method (strMC) or better than the investment into the index. It is worthwhile noting that the first two questions, i.e. "Is better than simMC?" and "Is better than strMC"?, are for the market capitalization methods complementary, which is indicated then by "--". The criterion for deciding which particular tracking portfolio is "better" is its terminal value at the end of the out-of-sample period.

Figure 1: Values of the tracking portfolios and of the S&P 500 Index over the out-of-sample period for the scenario SM with 20 equities pre-selected



Source: the authors.

Again, some general conclusions can be drawn from Tables 3 and 4, yet the results on mean active returns and information ratios reported for the sampling approaches to asset pre-selection should be interpreted with caution as they are affected by the diversity of measured outcomes of 100 random simulation runs. As it happens, the average values of mean (active) returns and information ratios displayed are marked with a presence of atypical and deviant values, which explains why they are occasionally large in magnitude. They should be attained reserved interpretation under each scenario. It is possible to gauge the separate impact of each one amongst the factors at play, having the others fixed and sliding along the results in the direction of the particular factor under scrutiny.

Before anything else, it must be observed that the market capitalization strategies are beyond question successful in outperforming the index. This is captured well by three independent indicators since in each case the mean active return is positive for either market capitalization approach to pre-selection and so is the information ratio. Also, each tracking portfolio ends up with a higher value than is the value of the underlying S&P 500 Index. When comparing the simple market capitalization approach with the stratified market capitalization approach, with the exception of when 75 assets is pre-selected, the simple market capitalization produces better results than the stratified market capitalization. This observations suggests that the most relevant factor for successful pre-selection may in fact be the share with which an asset is represented in the index that should be tracked. Its industrial affiliation may comparatively have a minor significance.

Table 3. Results for the scenarios MS, MM and ML

Scenario	Number of assets	20 assets				30 assets				40 assets			
	Preselection method	simMC	strMC	simSam	strSam	simMC	strMC	simSam	strSam	simMC	strMC	simSam	strSam
MS	Mean return (p.sep.)	0.0016	0.0009	-0.0003	-0.0003	0.0014	0.0013	-0.0002	-0.0002	0.0014	0.0006	-0.0003	-0.0004
	Information ratio (p.sep.)	0.24	0.15	-11.84	36.24	0.26	0.25	12.38	-15.03	0.29	0.12	371.03	-50.03
	Is better than simMC?	--	no	0%	1%	--	no	0%	0%	--	no	0%	1%
	Is better than strMC?	yes	--	7%	6%	yes	--	0%	1%	yes	--	8%	7%
	Is better than the index?	yes	yes	34%	37%	yes	yes	43%	35%	yes	yes	32%	23%
MM	Mean return (p.sep.)	0.0016	0.0009	-0.0004	-0.0002	0.0014	0.0013	-0.0003	-0.0002	0.0014	0.0006	-0.0003	-0.0003
	Information ratio (p.sep.)	0.24	0.15	53.12	31.83	0.26	0.25	0.83	-13.52	0.29	0.12	-10.84	21.87
	Is better than simMC?	--	no	0%	0%	--	no	1%	2%	--	no	0%	0%
	Is better than strMC?	yes	--	8%	7%	yes	--	2%	2%	yes	--	8%	5%
	Is better than the index?	yes	yes	45%	51%	yes	yes	40%	43%	yes	yes	28%	39%
ML	Mean return (p.sep.)	0.0016	0.0009	-0.0003	-0.0003	0.0014	0.0013	-0.0003	-0.0003	0.0014	0.0006	-0.0004	-0.0003
	Information ratio (p.sep.)	0.24	0.15	4.93	-46.52	0.26	0.25	2.55	59.63	0.29	0.12	-22.76	-14279
	Is better than simMC?	--	no	1%	1%	--	no	1%	2%	--	no	0%	0%
	Is better than strMC?	yes	--	7%	11%	yes	--	1%	2%	yes	--	9%	4%
	Is better than the index?	yes	yes	58%	53%	yes	yes	58%	57%	yes	yes	48%	51%
Scenario	Number of assets	50 assets				75 assets				100 assets			
	Preselection method	simMC	strMC	simSam	strSam	simMC	strMC	simSam	strSam	simMC	strMC	simSam	strSam
MS	Mean return (p.sep.)	0.0011	0.0003	-0.0003	-0.0004	0.0003	0.0005	-0.0004	-0.0004	0.0004	0.0003	-0.0004	-0.0004
	Information ratio (p.sep.)	0.25	0.07	-37.25	-4.54	0.07	0.15	-226.36	-5.66	0.12	0.10	-18.52	7.14
	Is better than simMC?	--	no	0%	0%	--	yes	12%	5%	--	no	5%	6%
	Is better than strMC?	yes	--	20%	15%	no	--	3%	1%	yes	--	8%	9%
	Is better than the index?	yes	yes	31%	25%	yes	yes	19%	22%	yes	yes	19%	23%
MM	Mean return (p.sep.)	0.0011	0.0003	-0.0005	-0.0004	0.0003	0.0005	-0.0004	-0.0004	0.0004	0.0003	-0.0005	-0.0003
	Information ratio (p.sep.)	0.25	0.07	-28.21	-8.37	0.07	0.15	7.43	-16.73	0.12	0.10	-26.61	-47.05
	Is better than simMC?	--	no	0%	0%	--	yes	12%	17%	--	no	3%	3%
	Is better than strMC?	yes	--	11%	16%	no	--	5%	4%	yes	--	8%	10%
	Is better than the index?	yes	yes	28%	30%	yes	yes	28%	29%	yes	yes	12%	19%
ML	Mean return (p.sep.)	0.0011	0.0003	-0.0004	-0.0004	0.0003	0.0005	-0.0005	-0.0004	0.0004	0.0003	-0.0005	-0.0004
	Information ratio (p.sep.)	0.25	0.07	-14.23	-20.90	0.07	0.15	-15.15	3.98	0.12	0.10	-4.57	-16.64
	Is better than simMC?	--	no	1%	0%	--	yes	9%	3%	--	no	2%	3%
	Is better than strMC?	yes	--	10%	11%	no	--	4%	1%	yes	--	3%	4%
	Is better than the index?	yes	yes	41%	44%	yes	yes	30%	31%	yes	yes	17%	18%

Source: the authors.

It seems that with the market capitalization approaches to asset pre-selection, there is little gain in adding more and more assets. Even with 20 assets, the index can be satisfactorily tracked and a higher number of assets may be counterproductive. Albeit with the sampling approaches the results are more dispersed and less compact, the picture is apparently similar and the pattern is that with a higher number of assets pre-selected there is a higher tendency to fail in performance. The percentage of simulation runs (which is just a snapshot of actual opportunities) when the index is outperformed in terms of terminal value tends to fall with increasing the number of pre-selected assets. In addition, these percentages are not convincing in comparison to the unanimous "yes" for the market capitalization strategies that in each case wound up with better terminal values than the index did. The percentages of outperformance relative to the index were recorded only the ML and LM scenarios (20 and 30 assets) as well as for the MM scenario (20 assets).

Tracking portfolios counting 20, 30 and 40 assets fare fairly well, which is manifested in higher values of their information ratios, though at the same time accompanied by small mean active returns. This also signifies that in these situations tracking error volatilities (that appear in the denominator of information ratios) are suppressed and small. By adding other assets the performance of tracking portfolios decreases as is the situation for portfolios of 50, 75 and

100 assets in the case of which the tracking error volatility decreases, yet together with the mean active return that decreases at a higher pace. The described behaviour reflected in the decline of the mean active return in reaction to extra assets added to the tracking portfolio is caused by induced transaction costs. Transaction costs arise in connection with the number of assets regardless of whether a scenario of small, moderate or large transaction costs is considered and regardless of the amount of the budged available.

Table 4. Results for the scenarios SM, MM and LM

Scenario	Number of assets	20 assets				30 assets				40 assets			
	Preselection method	simMC	strMC	simSam	strSam	simMC	strMC	simSam	strSam	simMC	strMC	simSam	strSam
SM	Mean return (p.sep.)	0.0016	0.0009	-0.0003	-0.0003	0.0014	0.0013	-0.0003	-0.0002	0.0014	0.0006	-0.0004	-0.0004
	Information ratio (p.sep.)	0.24	0.15	1.30	-18.61	0.26	0.25	26.77	60.18	0.29	0.12	18.48	-15.69
	Is better than simMC?	--	no	1%	0%	--	no	0%	1%	--	no	0%	0%
	Is better than strMC?	yes	--	12%	7%	yes	--	0%	2%	yes	--	6%	8%
	Is better than the index?	yes	yes	29%	25%	yes	yes	17%	23%	yes	yes	7%	12%
MM	Mean return (p.sep.)	0.0016	0.0009	-0.0004	-0.0002	0.0014	0.0013	-0.0003	-0.0002	0.0014	0.0006	-0.0003	-0.0003
	Information ratio (p.sep.)	0.24	0.15	53.12	31.83	0.26	0.25	0.83	-13.52	0.29	0.12	-10.84	21.87
	Is better than simMC?	--	no	0%	0%	--	no	1%	2%	--	no	0%	0%
	Is better than strMC?	yes	--	8%	7%	yes	--	2%	2%	yes	--	8%	5%
	Is better than the index?	yes	yes	45%	51%	yes	yes	40%	43%	yes	yes	28%	39%
LM	Mean return (p.sep.)	0.0016	0.0009	-0.0001	-0.0004	0.0014	0.0013	-0.0002	-0.0003	0.0014	0.0006	-0.0004	-0.0003
	Information ratio (p.sep.)	0.24	0.15	13.33	1.43	0.26	0.25	-23.55	-0.26	0.29	0.12	20.97	15.57
	Is better than simMC?	--	no	1%	0%	--	no	2%	3%	--	no	0%	0%
	Is better than strMC?	yes	--	15%	4%	yes	--	2%	4%	yes	--	4%	8%
	Is better than the index?	yes	yes	57%	38%	yes	yes	51%	46%	yes	yes	37%	41%
Scenario	Number of assets	50 assets				75 assets				100 assets			
	Preselection method	simMC	strMC	simSam	strSam	simMC	strMC	simSam	strSam	simMC	strMC	simSam	strSam
SM	Mean return (p.sep.)	0.0011	0.0003	-0.0004	-0.0003	0.0003	0.0005	-0.0004	-0.0005	0.0004	0.0003	-0.0004	-0.0004
	Information ratio (p.sep.)	0.25	0.07	-82.52	-12.84	0.07	0.15	10.80	-1.10	0.12	0.10	-6.56	77.54
	Is better than simMC?	--	no	0%	0%	--	yes	9%	9%	--	no	4%	6%
	Is better than strMC?	yes	--	20%	18%	no	--	4%	1%	yes	--	7%	8%
	Is better than the index?	yes	no	1%	0%	no	no	0%	0%	no	no	0%	0%
MM	Mean return (p.sep.)	0.0011	0.0003	-0.0005	-0.0004	0.0003	0.0005	-0.0004	-0.0004	0.0004	0.0003	-0.0005	-0.0003
	Information ratio (p.sep.)	0.25	0.07	-28.21	-8.37	0.07	0.15	7.43	-16.73	0.12	0.10	-26.61	-47.05
	Is better than simMC?	--	no	0%	0%	--	yes	12%	17%	--	no	3%	3%
	Is better than strMC?	yes	--	11%	16%	no	--	5%	4%	yes	--	8%	10%
	Is better than the index?	yes	yes	28%	30%	yes	yes	28%	29%	yes	yes	12%	19%
LM	Mean return (p.sep.)	0.0011	0.0003	-0.0003	-0.0005	0.0003	0.0005	-0.0004	-0.0004	0.0004	0.0003	-0.0004	-0.0004
	Information ratio (p.sep.)	0.25	0.07	3.63	-14.19	0.07	0.15	-90.22	-10.23	0.12	0.10	14.96	-8.67
	Is better than simMC?	--	no	1%	0%	--	yes	9%	9%	--	no	7%	3%
	Is better than strMC?	yes	--	15%	12%	no	--	2%	4%	yes	--	13%	7%
	Is better than the index?	yes	yes	44%	34%	yes	yes	38%	42%	yes	yes	31%	26%

Source: the authors.

In computations and preparing graphical presentations, the software R version 3.0.1 (R Core Team, 2013) was employed with several of its libraries, `fPortfolio`, `quadprog` and `timeSeries`.

5. Conclusion

The paper concerns itself with comparing four strategies to asset pre-selection in index tracking applied to the S&P 500 Index. The performance behaviour in several scenarios of two market capitalization approaches is compared with, and assessed against, that of two random sampling strategies. With this it must be admitted and disclosed that the motivation is rather heuristic. There are no guidelines laid down how to pre-select assets, although it is a conventional practice to combine the market capitalization criterion with the past performance of assets or common sense. Owing to the superior position of the random sampling method of selection in statistics, it might be possible that this approach might be relevant also in the task of portfolio selection. All the same, this is not found to be the case and the market capitalization criterion fares better and is to be recommended without stipulation – even with its simple variant and not with any room given to the industrial affiliation of individual assets. In the comparison of pre-selection approaches, several other factors are given also evaluation, viz. the number of assets and the size of transaction costs in relation to the budget available.

The investigation shows that tracking portfolios composed of assets pre-selected by random sampling are prone to give considerably worse results when confronted with the performance of the tracked index. In the case of the two sampling approaches this fact is proven by the percentage of simulation runs in which the tracking portfolio outperforms the index or performs better than the tracking portfolios resulting from the market capitalization criterion. These percentage statistics are computed for a total of 100 simulations.

All the tracking portfolios constructed using the market capitalization approaches yielded better performance that the index did when evaluated at the end of the out-of-sample period by a mere comparison of terminal values. This is clearly readable from Tables 3 and 4 and might be more aptly visualized as in Figure 1. Yet, there is no benefit in producing such a graph as is shown in Figure 1 for each scenario or setting since they are very alike and virtually coincide with the information communicated. If the ambition is to outperform the index, the simple market capitalization criterion is preferable to the stratified one as is recorded in higher mean active returns and information ratios. This corresponds to the findings obtained by the empirical studies by Alexander and Dimitriu (2005) and by Rey and Seiler (2001).

One of the factors investigated is the impact of the number of assets upon the performance of a tracking portfolio. The results indicate that tracking portfolios composed out of 20, 30 and 40 assets exhibit satisfactory performance as measured by the information ratio, though at the expense of relatively low mean active returns. Adding more assets to the tracking portfolio seems to be of no avail as is apparent in the performance of tracking portfolios of 50, 75 and 100 assets. This is but in full compatibility with the observation of Ray and Seiler (2001) who concluded that a higher number of assets in the tracking portfolio pushes the tracking error down, but it is not as significant as it might be expected a priori by theoretical reasoning or through economic intuition.

Eventually, it appears that neither the amount of budget available nor the size of fixed and variable transaction costs (if kept in reasonable relations) is a factor of major significance. Their impact upon the performance of tracking portfolios is marginal and their higher size is not a difficulty in outperforming the index. What is possibly more relevant is how the holdings of the tracking portfolio are determined. They are in the paper elicited by means of quadratic error tracking, which is a standard in passive portfolio management.

The testing and performance evaluation of the 4×6 asset selection strategies (differentiated chiefly by the pre-selection method itself and then by the number of assets for tracking portfolio optimization) was accomplished by inspecting the performance of tracking portfolios during the out-of-sample period; yet, these tracking portfolios were built and optimized using historical data of the in-sample period. For this reason, it only can be anticipated that these strategies would fare equally well also in the in-sample period. The question of their continuing validity (or rather their poor or satisfactory performance) in a near or distant future might be examined by studying the behaviour of the selection strategies in out-of-sample periods of varying length. Still, it is possible to speculate further that prolongation of the out-of-sample period would overall lead to increased (out-of-sample) tracking error and that for portfolios resulting from asset pre-selection under the criterion of market capitalization this tracking error would be smaller in comparison to tracking portfolios selected by a different criterion. The finding that the best asset pre-selection method is the one based on (simple) market capitalization is difficult to state with a general authority as the performance of any strategy depends on a number of factors, the type of the benchmark not excluding. The method of simple market capitalization would perhaps produce tracking portfolios with satisfactory performance also in the case if a capitalization-weighted (i.e. market-value-weighted) index would serve the role of the benchmark. On the contrary, this pre-selection method may easily fail in the future if the underlying benchmark index changes its composition and the alteration affects the large-capitalization assets representing a considerable share on its value.

All in all, for asset pre-selection, the market capitalization approach is recommendable in comparison to the random sampling approach. Moreover, simple asset pre-selection based on market capitalization is preferable over stratified asset pre-selection of such. Although the results obtained are to some degree authoritative and compelling, in the paper only transaction costs are considered that are directly incorporated in the optimization process. Transaction costs depend on the number of traded and held assets, and it appears advisable to relate a good part of transaction cost to the bid-ask spread of individual assets. The reason is that – as a matter of fact – (il)liquidity costs are somewhat ignored in the paper. These depend right on the bid-ask spread, which may prove itself to be a determinant worthy of further examination. The discussed line of research is only an inspiration for the future.

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